Biased-noise cat qubits

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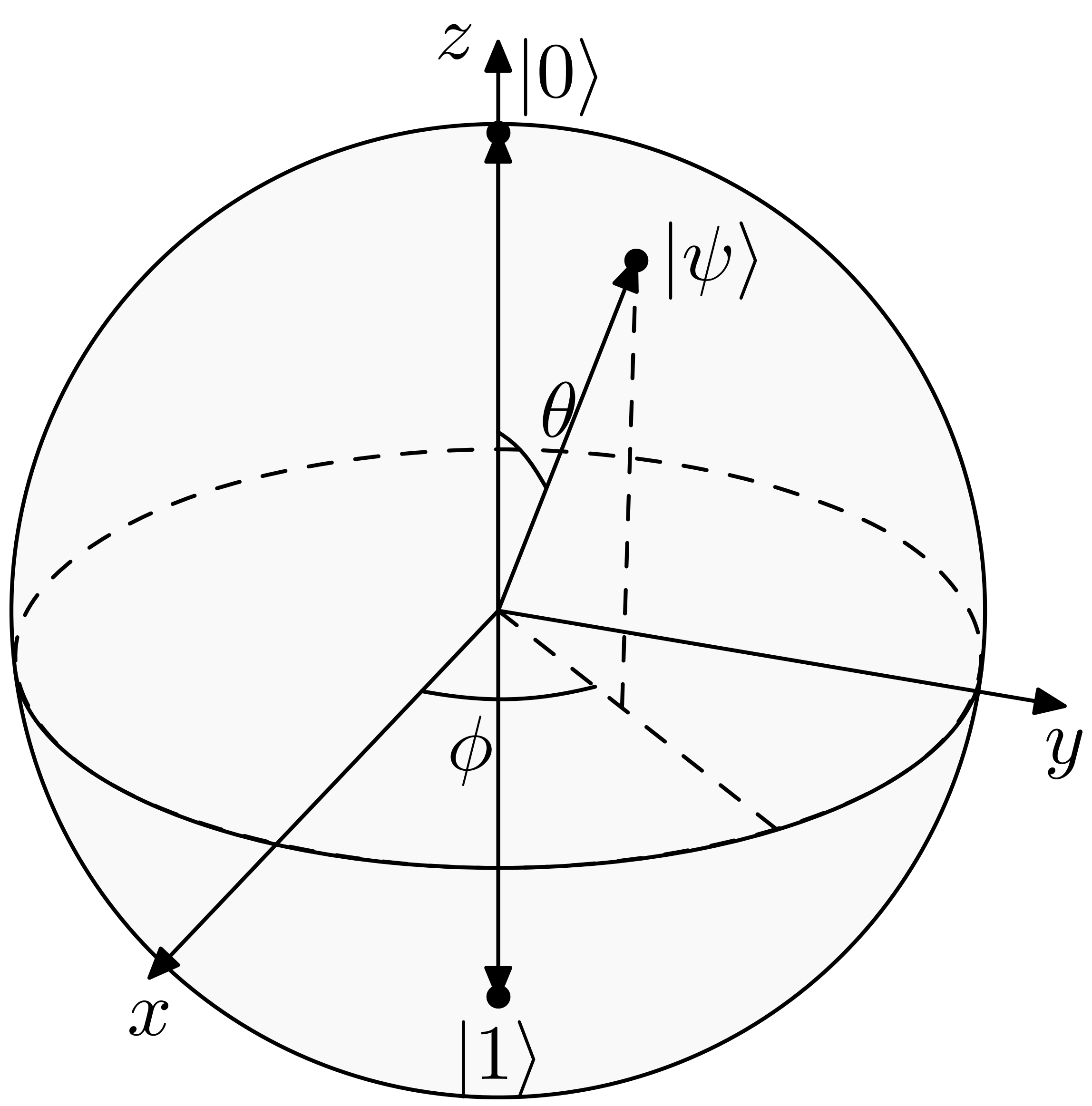
From water tank to quantum box (25 years with Pierre)

- $\psi(x,t)\rangle$.

Quantum bit (qubit)



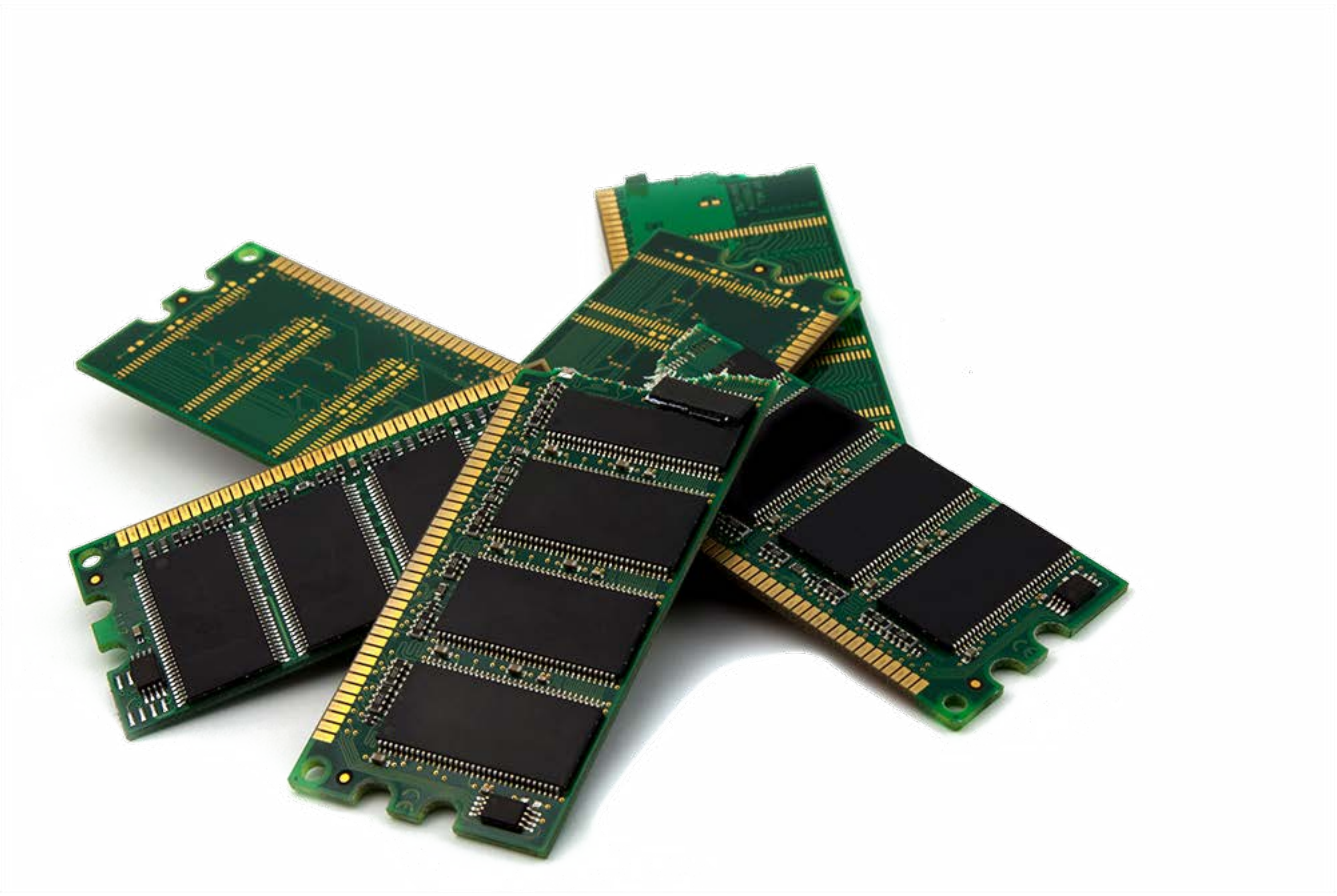
- Pauli operators

$$\mathbf{X} = \sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0| = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathbf{Z} = \sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

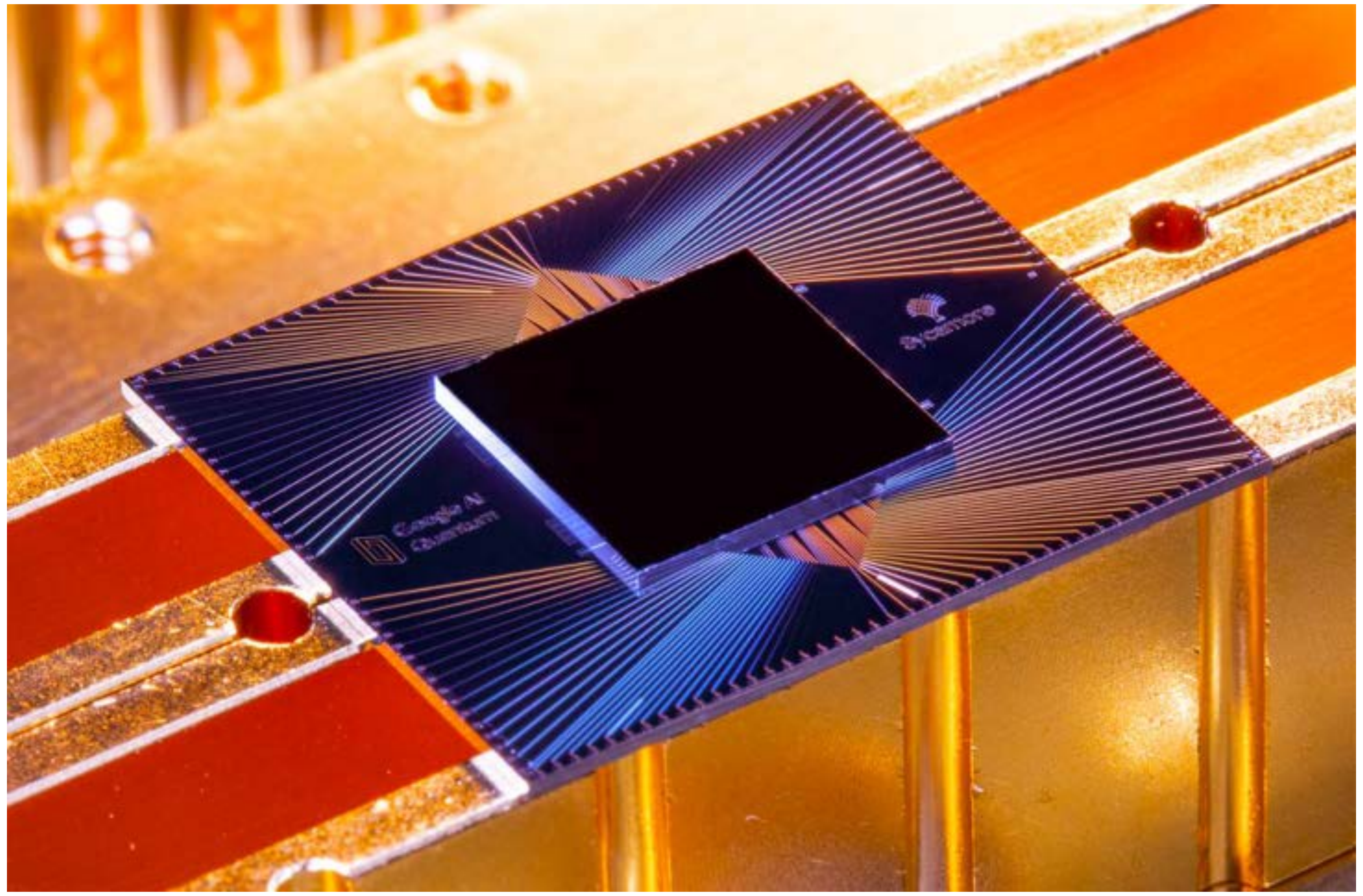
$$\mathbf{Y} = \sigma_y = i\sigma_x\sigma_z$$

Quantum hardware is (too) noisy



Classical RAM (Random Access Memory)

$\sim 10^{-25}$ errors per bit per operation

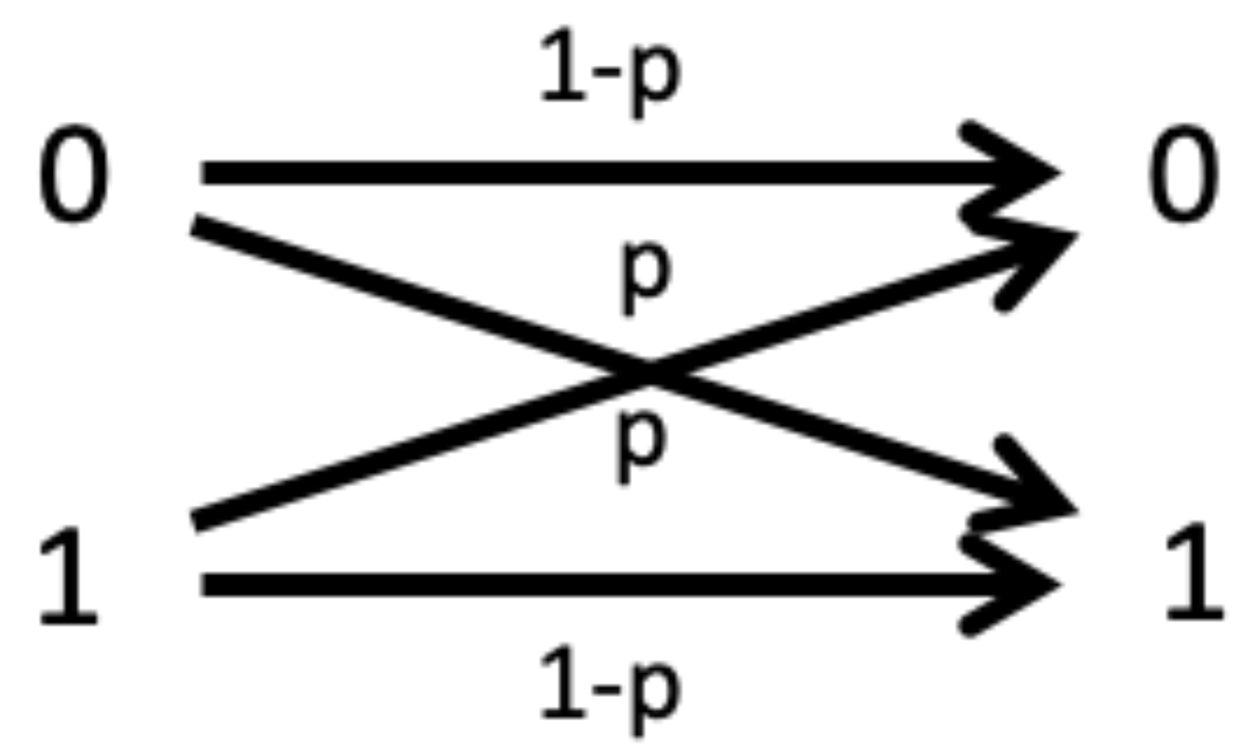


Quantum processor

$\sim 10^{-3} - 10^{-4}$ errors per bit per operation

Large scale quantum computation requires $\sim 10^{-10} - 10^{-15}$

Classical error correction



- Classical noise: bit-flip errors

- Idea of error correction: redundancy

$$0_L \to 000 \quad \text{and} \quad 1_L \to 111$$

- 1-bit errors tractable by majority vote

- Probability of failure: 2 or 3 errors.

$$p_L = 3p^2(1-p) + p^3$$

- For p<1/2 $\qquad p_L < p$

Quantum error correction: main issues

- How to detect errors without destroying quantum information? Measuring a qubit projects its state:

$$c_0|0\rangle + c_1|1\rangle \to 0 \text{ or } 1$$

- Errors are continuous: an error is not necessarily of the form $|\psi\rangle \to \sigma_x|\psi\rangle$.

  For instance, we can also have $|\psi\rangle \to e^{i\theta\sigma_x}|\psi\rangle \qquad \theta \in [0, 2\pi)$

- Errors are not only bit-flips or unitaires generated by σ_x ? How to model errors?

1- How to detect errors without destroying information?

Objective: protect any superposition $c_0|0\rangle + c_1|1\rangle$ without knowledge of coefficients.

Tool: repetition by entanglement $c_0|0\rangle + c_1|1\rangle \rightarrow c_0|000\rangle + c_1|111\rangle$

Detection: parity checks by measuring $Z_1Z_2 = \sigma_z \otimes \sigma_z \otimes I$ and $Z_2Z_3 = I \otimes \sigma_z \otimes \sigma_z$

Quantum error correction: main issues

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- Errors are not only bit-flips or unitaires generated by σ_x ? How to model errors?

2 and 3: How to model errors? What is error correction?

<u>Language of open quantum systems:</u>

State space:

$$|\psi\rangle \in \mathcal{H} \longrightarrow \rho \in \{\xi \in \mathcal{B}_1(\mathcal{H}) \mid \rho^\dagger = \rho, \rho \geq 0, \mathrm{Tr}(\rho) = 1\}$$

Particular case of a pure quantum state:

$$\rho = \Pi_{|\psi\rangle} = |\psi\rangle\langle\psi|$$

2 and 3: How to model errors? What is error correction?

Quantum noise: interaction with environment

System+environment $\in \mathcal{H}_s \otimes \mathcal{H}_{\mathrm{env}}$

$$\mathcal{E}(\rho_s) = \boldsymbol{tr}_{\mathrm{env}}\left[\boldsymbol{U}_\tau(\rho_s \otimes \rho_{\mathrm{env}})\boldsymbol{U}_\tau^\dagger\right] = \sum_k \boldsymbol{E}_k \rho_s \boldsymbol{E}_k^\dagger \quad \text{with} \quad \sum_k \boldsymbol{E}_k^\dagger \boldsymbol{E}_k = \boldsymbol{I}.$$

Quantum error correction in a nutshell:

Code space: $\mathcal{H}_c \subset \mathcal{H}_s$

Like errors, the error correction (measurement and feedback) can be modelled as a quantum operation:

$$\rho \to \mathcal{R}(\rho) = \sum_k \boldsymbol{R}_k \rho \boldsymbol{R}_k^\dagger$$

This corrects an error channel $\rho \mapsto \mathcal{E}(\rho)$ if $\forall \rho_c \in \mathcal{H}_c, \quad \mathcal{R} \circ \mathcal{E}(\rho_c) = \rho_c.$

Why does quantum error correction work?

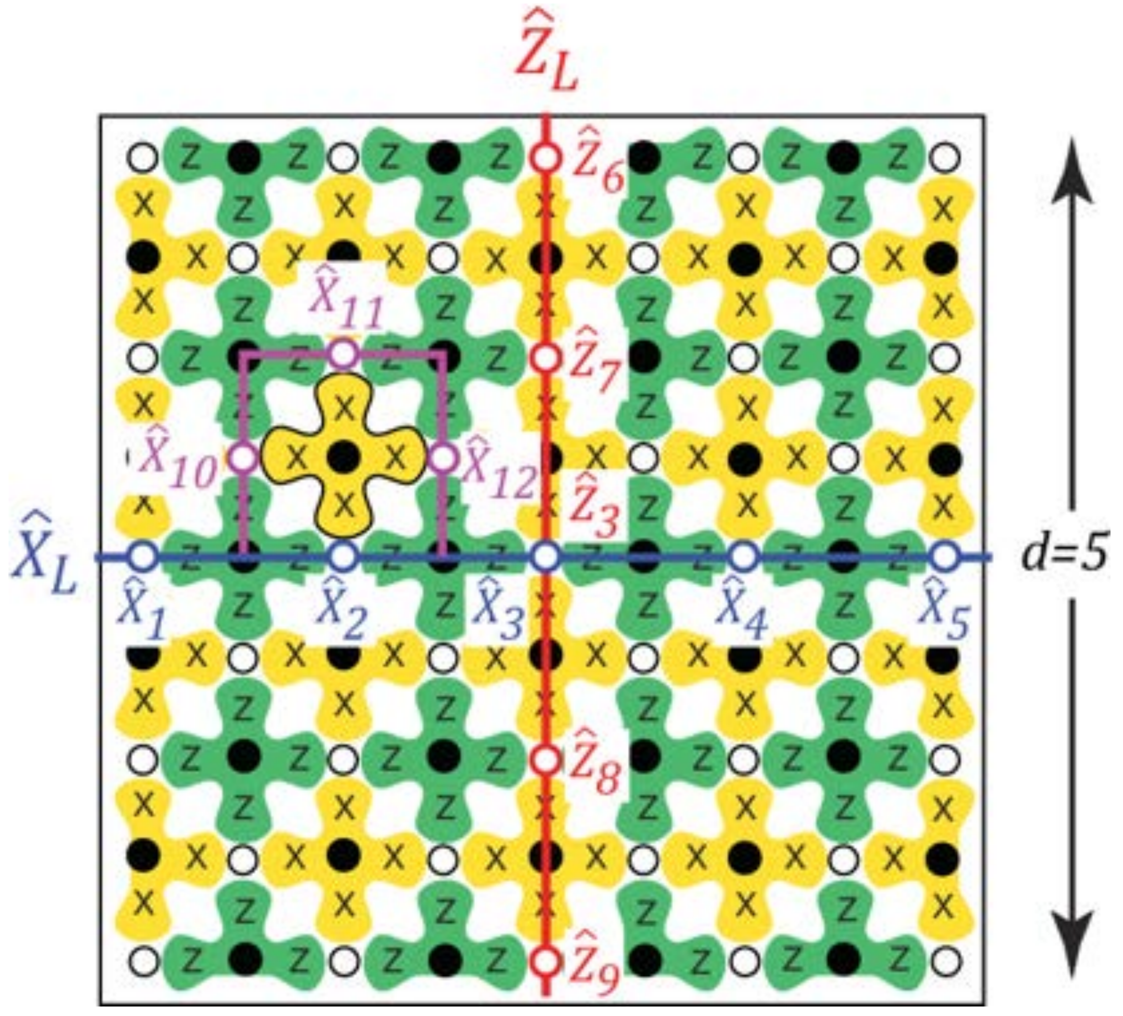
<u>Theorem:</u> discretisation of error channels

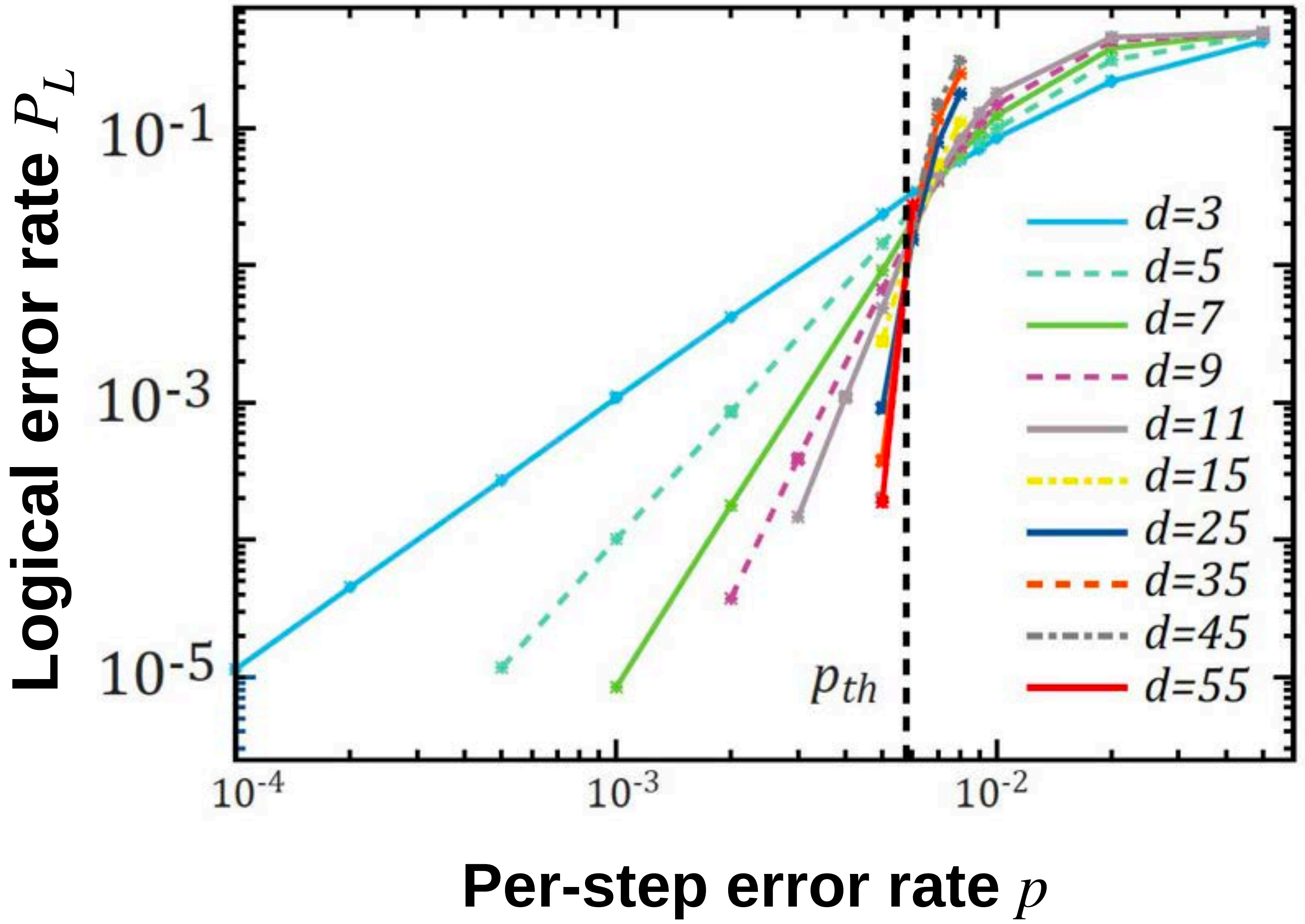
If the operation $\mathscr{R}$ corrects the error channel $\mathscr{E}$, it corrects any other error channel $\mathscr{F}$ whose elements F_k are complex linear combinations of the elements E_k of $\mathscr{E}$:

$$\mathcal{R} \circ \mathcal{E}(\rho_c) = \rho_c \qquad \Rightarrow \qquad \mathcal{R} \circ \mathcal{F}(\rho_c) = \rho_c .$$

<u>Corollary:</u> it suffices to correct the operations $\{I, \sigma_x, \sigma_z, \sigma_y = i\sigma_x\sigma_z\}$ to correct any single qubit errors.

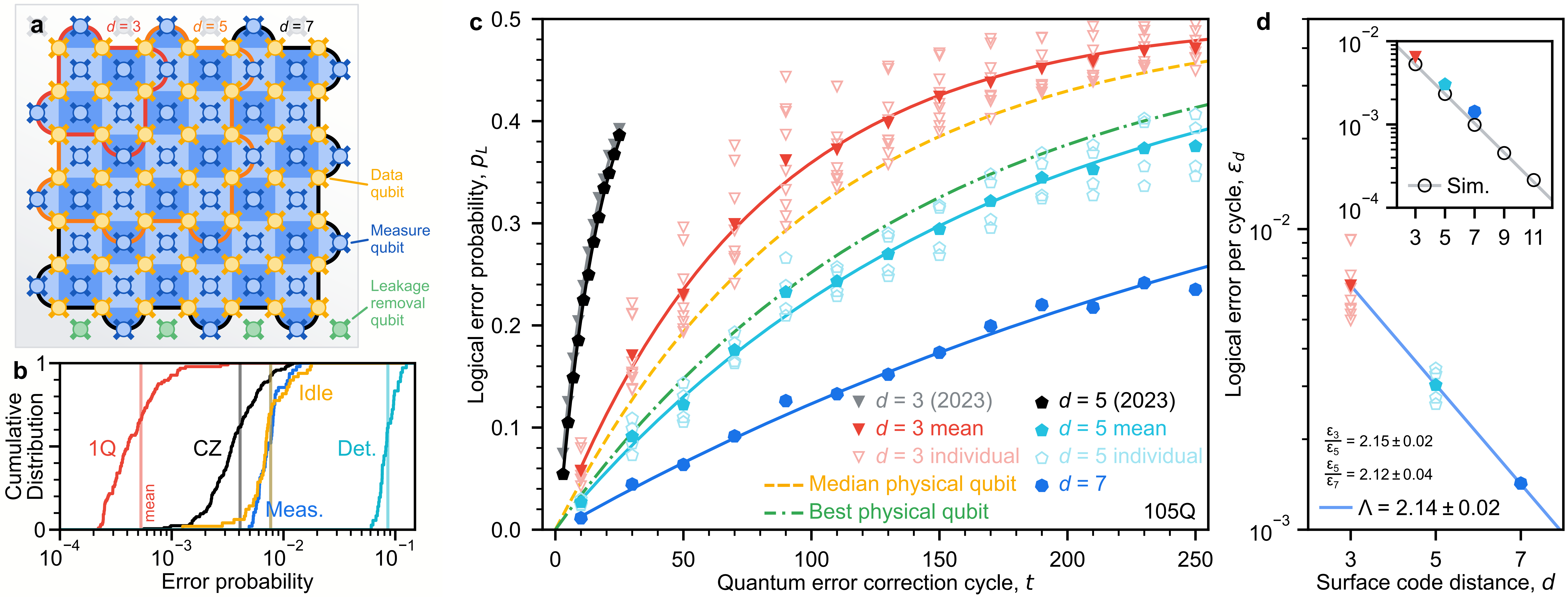
Cost of error correction with surface code architecture





Figures borrowed from A. Fowler et al., Phys. Rev. A 86, 2012.

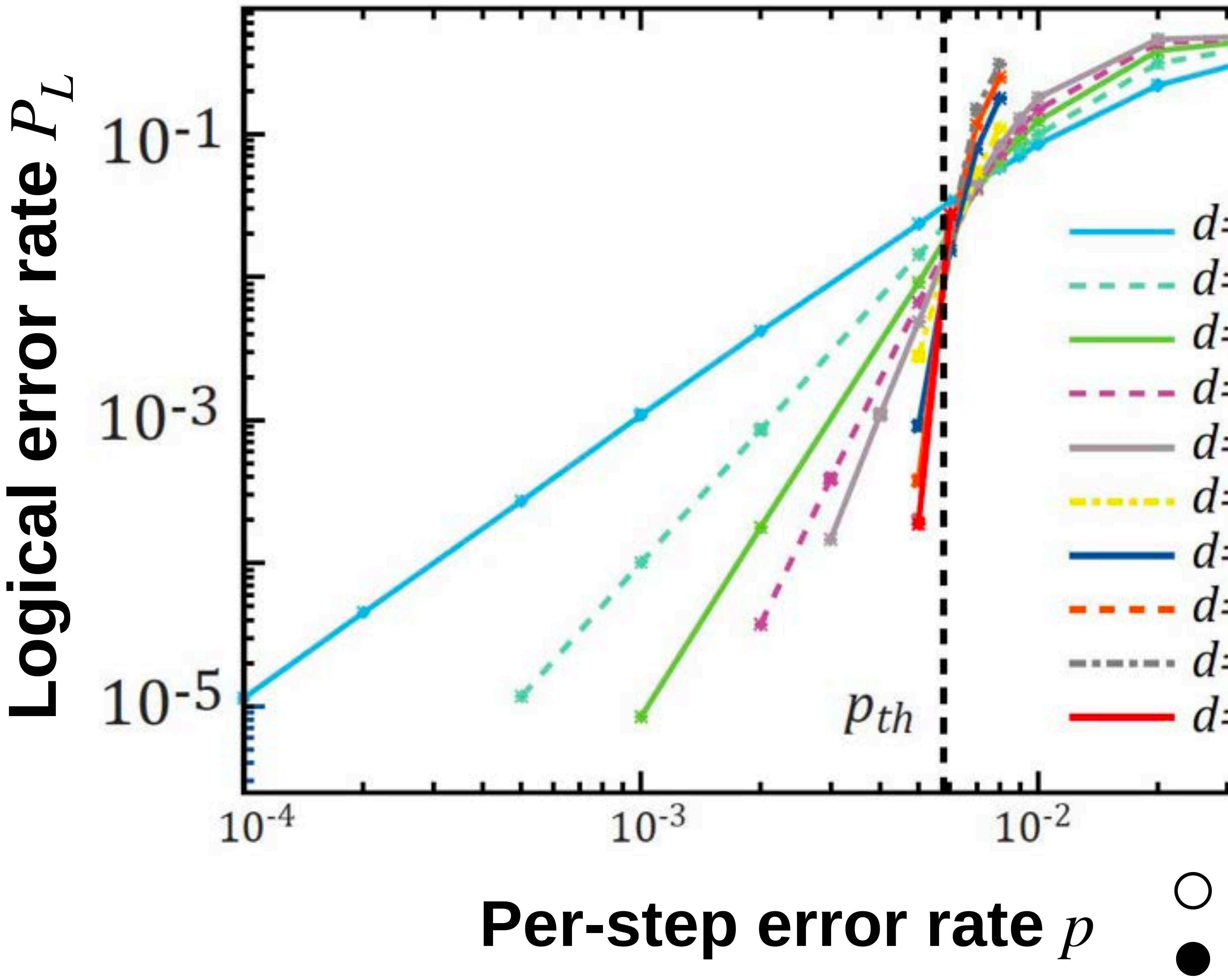
State of progress



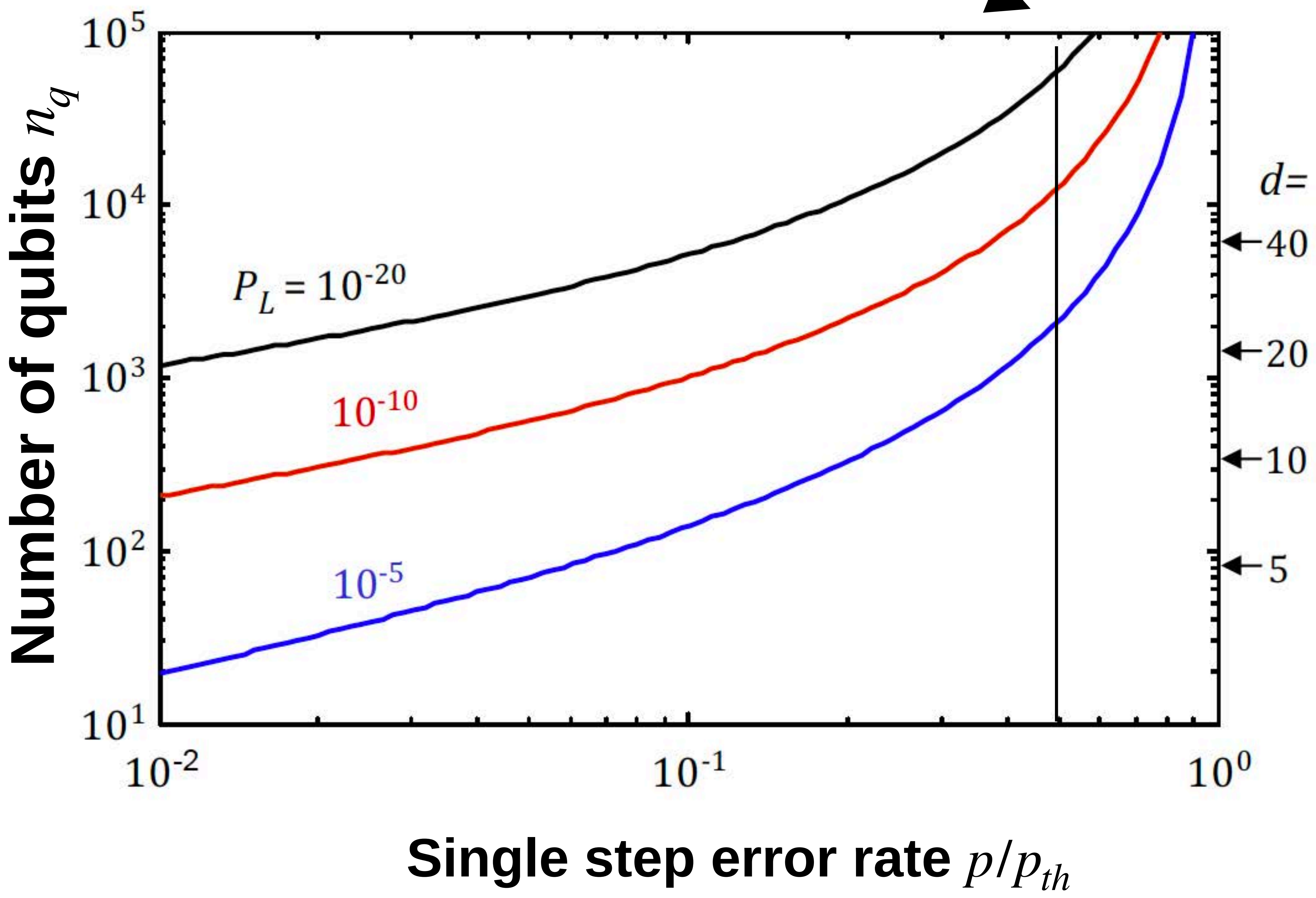
$$p_L \propto \left(\frac{p}{p_{\text{th}}}\right)^{\frac{d+1}{2}} \qquad \Gamma := (p/p_{\text{th}})$$

Google Quantum AI, Nature, 2025.

State of progress



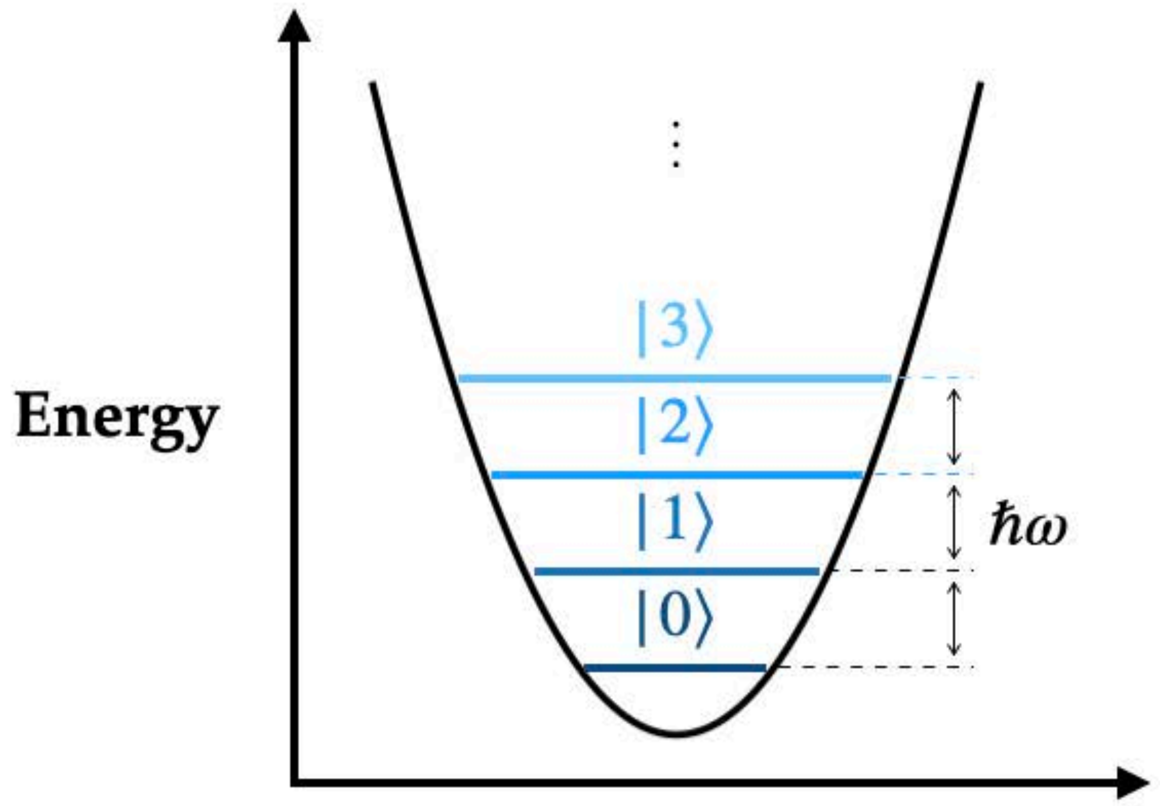
Google Quantum AI, Nature, 2025



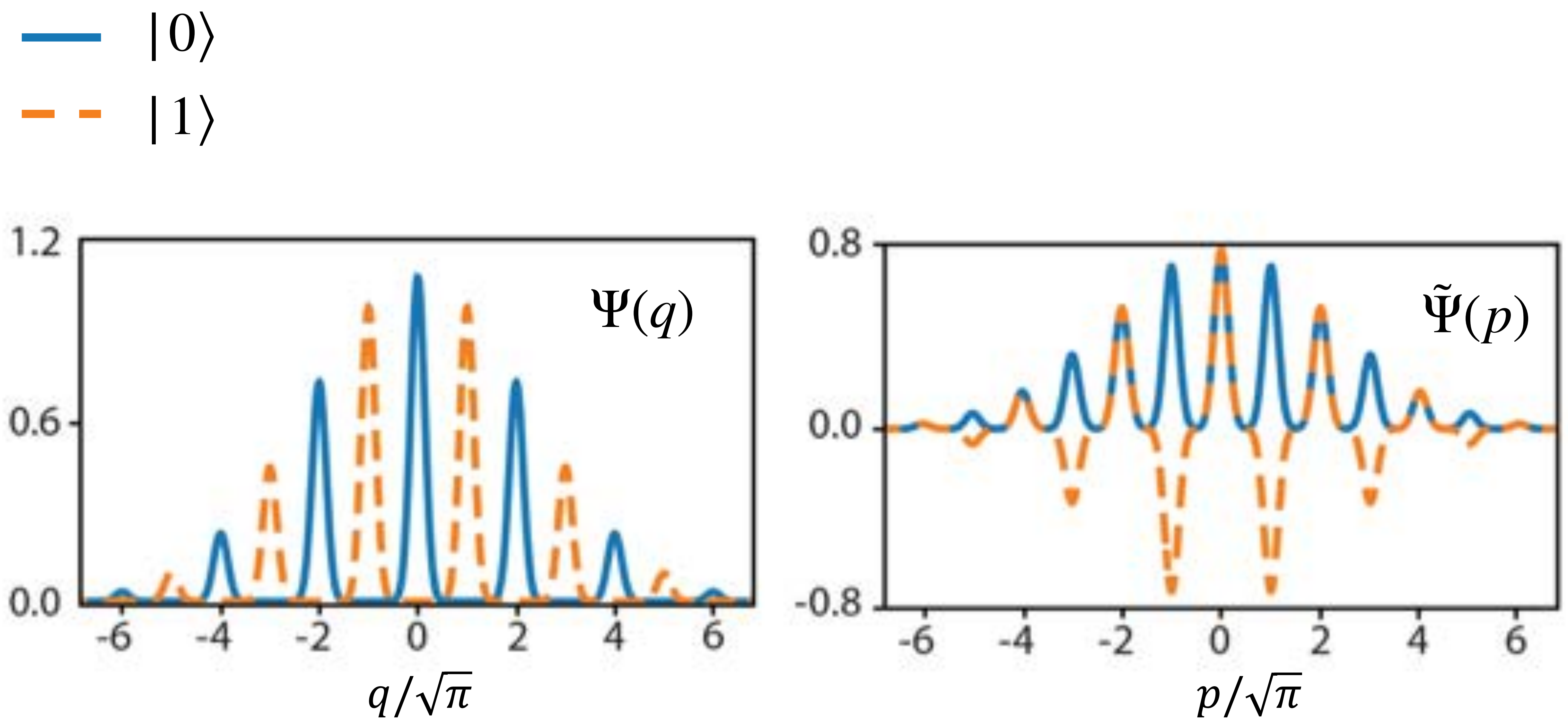
A. Fowler et al., Phys. Rev. A 86, 2012.

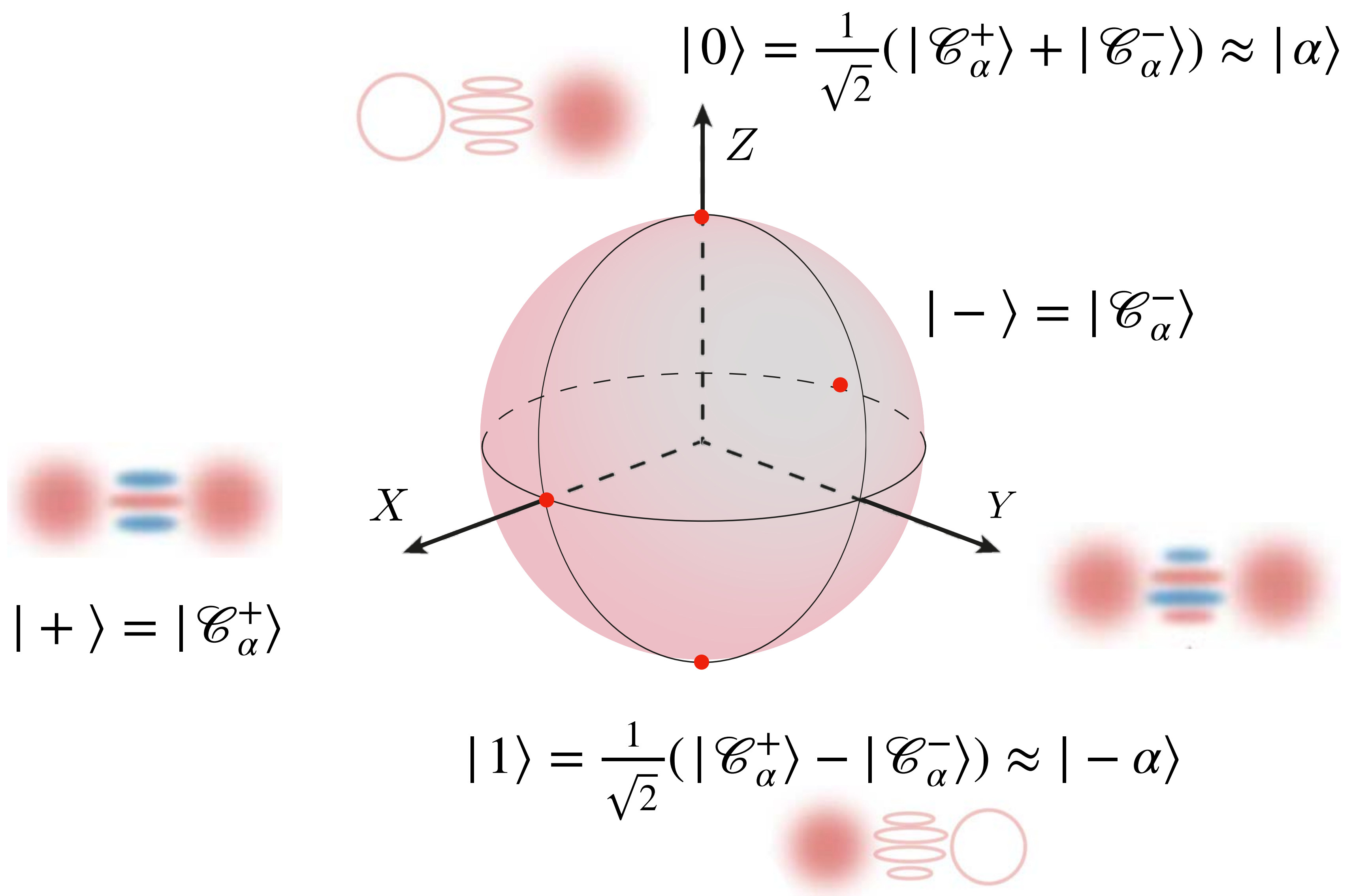
Shortcut: QEC with bosonic codes

A. Joshi, K. Noh, Y. Gao,
Quantum Sci. Technol. 6 033001 (2021)



$$\mathscr{H} = \text{span}\{\,|n\rangle, n \in \mathbb{N}\}$$



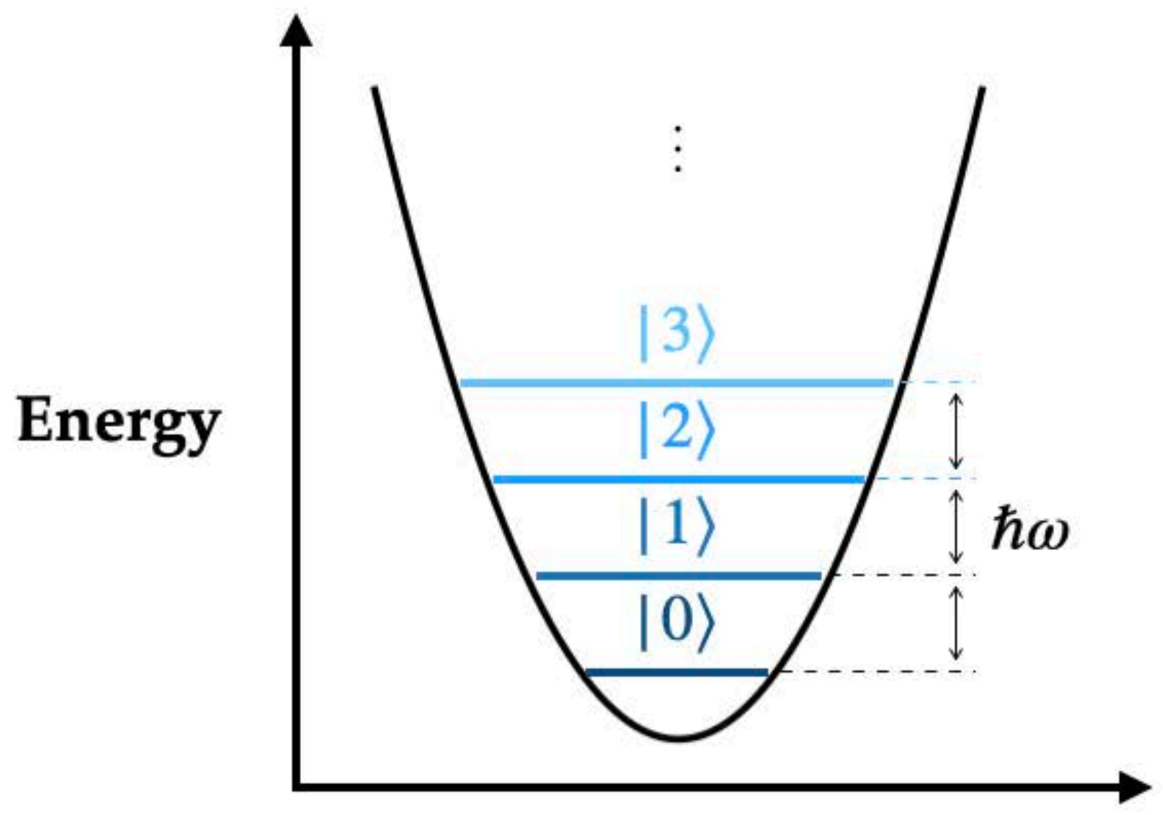


D. Gottesman, A. Kitaev, J. Preskill, Phys. Rev. A 64, 2001.

P.T. Cochrane *et al.*, Phys. Rev. A,1999.
Z. Leghtas *et al.*, PRL, 2013.

Quantum harmonic oscillator

Encode information in the infinite dimensional Hilbert space of a single quantum harmonic oscillator



Fock states: $\mathscr{H} = \text{span}\{|n\rangle, n \in \mathbb{N}\}$

$$|n\rangle = \psi_n(x) = (\frac{1}{\pi})^{1/4} \frac{1}{\sqrt{2^n n!}} e^{-x^2/2} H_n(x), \qquad H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \quad \textbf{and} \quad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\hat{H} = -\frac{\hbar\omega}{2}\frac{\partial^2}{\partial x^2} + \frac{\hbar\omega}{2}x^2 = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$$

$$\hat{a} = \frac{1}{\sqrt{2}}(x + \frac{\partial}{\partial x}) \quad , \quad \hat{a}^\dagger = \frac{1}{\sqrt{2}}(x - \frac{\partial}{\partial x})$$

Coherent states: $\mathscr{H} = \text{span}\{|\alpha\rangle, \alpha \in \mathbb{C}\}$

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n \in \mathbb{N}} \frac{\alpha^n}{\sqrt{n!}} |n\rangle = \frac{1}{\pi^{1/4}} e^{i\sqrt{2}x\Im(\alpha)} e^{-\frac{(x-\sqrt{2}\Re(\alpha))^2}{2}}$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

Quantum harmonic oscillator

State space: $\{\rho \in \mathscr{B}_1(\mathscr{H}) \mid \rho^\dagger = \rho,\ \rho \geq 0,\ \mathrm{tr}(\rho) = 1\}$ $\qquad \mathscr{H} = \mathrm{span}\{\,|n\rangle, n \in \mathbb{N}\}$

Dynamics of driven damped harmonic oscillator:

$$\frac{d\rho}{dt} = -i[H,\rho] + L\rho L^\dagger - \frac{1}{2}L^\dagger L\rho - \frac{1}{2}\rho L^\dagger L \qquad H = i(\epsilon_1 \hat{a}^\dagger - \epsilon_1^* \hat{a}) \qquad L = \sqrt{\kappa_1}\hat{a}$$

Equivalent formulation: Wigner function $\qquad W^\rho(x,p) = \frac{2}{\pi}\mathrm{tr}\left(\rho \mathbf{D}_\alpha e^{i\pi\hat{a}^\dagger\hat{a}}\mathbf{D}_{-\alpha}\right)$

$\alpha = x + ip \qquad \mathbf{D}_\alpha = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ $\qquad$ Coherent state: $|\alpha\rangle = \mathbf{D}_\alpha|0\rangle$

$$\frac{\partial}{\partial t}W^\rho = \frac{\kappa}{2}\left(\frac{\partial}{\partial\alpha}(\alpha - \bar{\alpha}) + \frac{\partial}{\partial\alpha^*}(\alpha^* - \bar{\alpha}^*) + \frac{\partial^2}{\partial\alpha\partial\alpha^*}\right)W^{(\rho)} \qquad \bar{\alpha} = \frac{2\epsilon_1}{\kappa_1} := \bar{x} + i\bar{p}$$

$$\lim_{t\to\infty} W_t^\rho(x,p) = \frac{2}{\pi}\exp\left(-2((x-\bar{x})^2 + (p-\bar{p})^2)\right) = W^{|\bar{\alpha}\rangle\langle\bar{\alpha}|}(x,p)$$

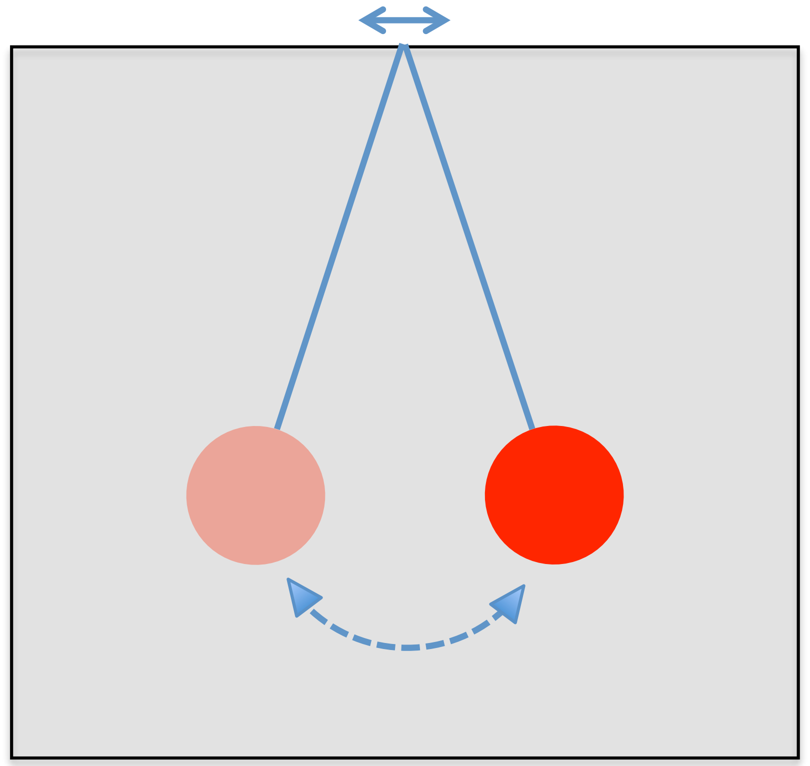
Possible shortcuts: autonomous QEC with bosonic codes

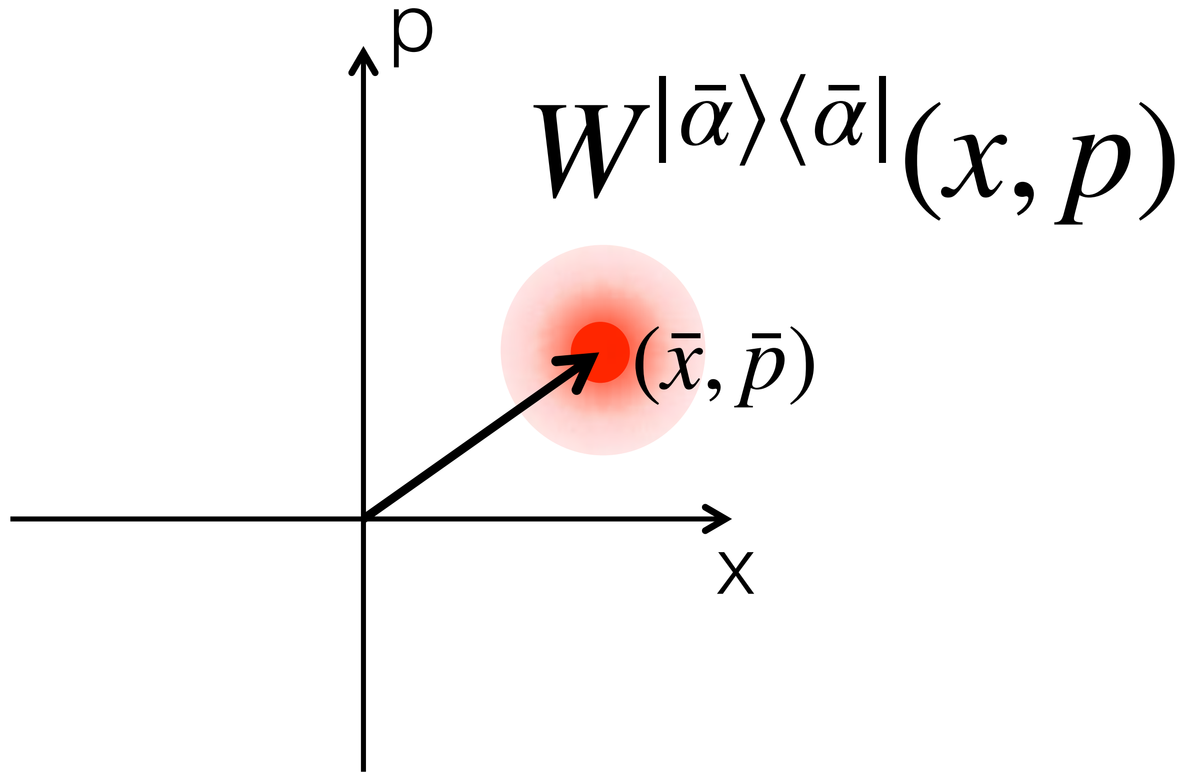
« Single-photon » driven-damped harmonic oscillator

$$\bar{\alpha} = \frac{2\epsilon_1}{\kappa_1} := \bar{x} + i\bar{p}$$

$$\frac{d\rho}{dt} = -i[H,\rho] + L\rho L^\dagger - \frac{1}{2}L^\dagger L\rho - \frac{1}{2}\rho L^\dagger L$$

$$H = i(\epsilon_1 \hat{a}^\dagger - \epsilon_1^* \hat{a}) \qquad L = \sqrt{\kappa_1}\hat{a}$$



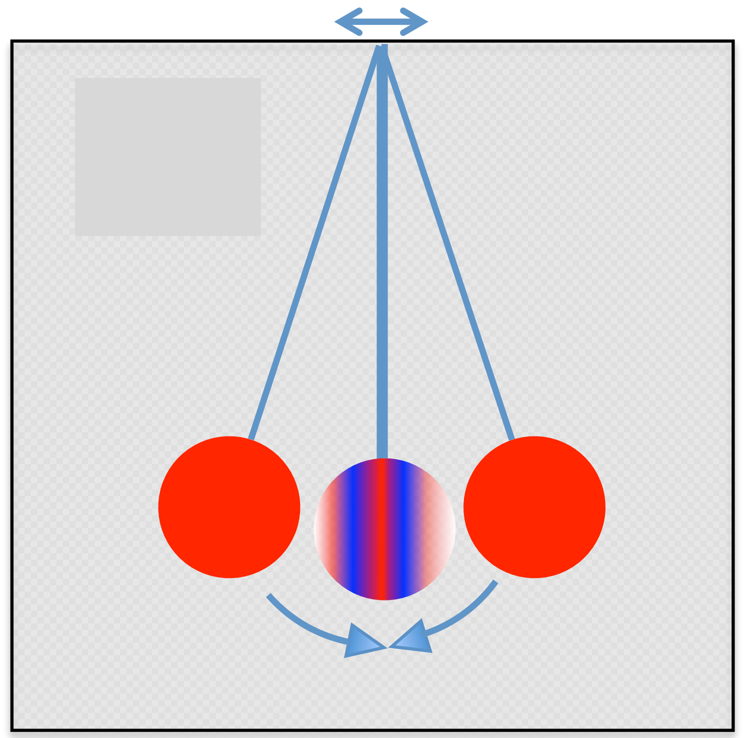


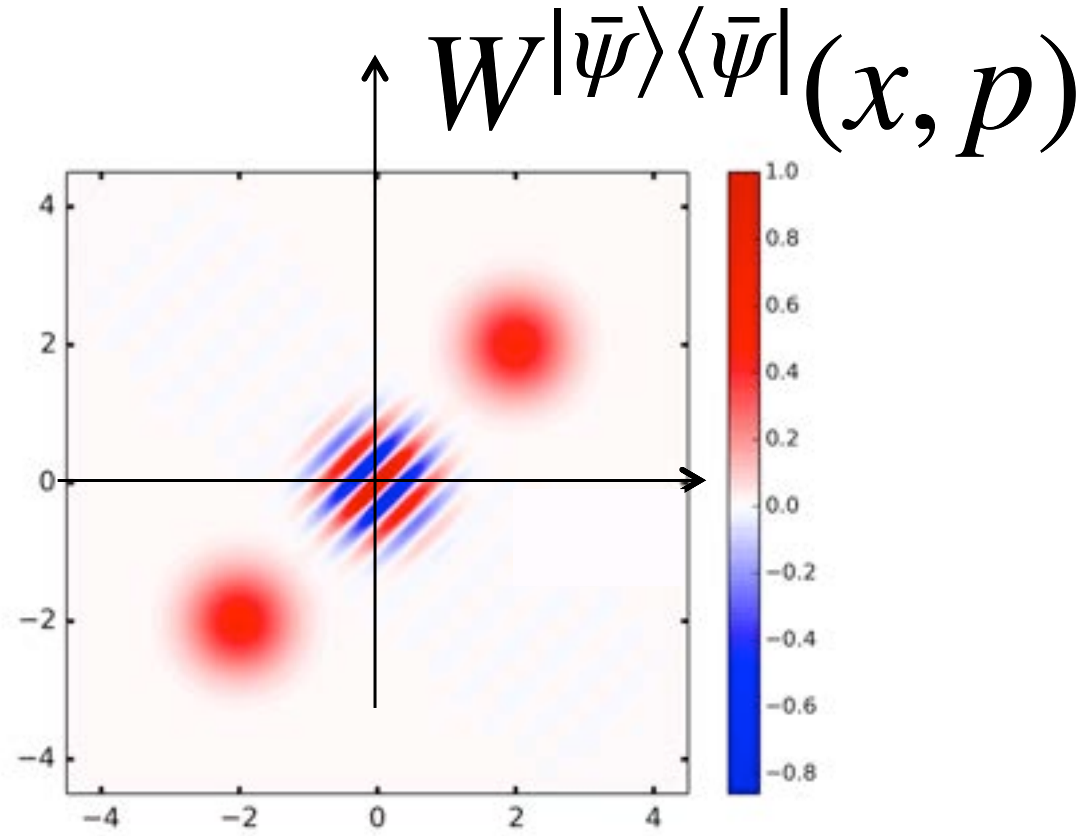
« Two-photon » driven-damped harmonic oscillator

$$H = i(\epsilon_2 \hat{a}^{\dagger 2} - \epsilon_2^* \hat{a}^2) \qquad L = \sqrt{\kappa_2}\hat{a}^2$$

$$\rho(t) \to \mathscr{B}_1\left(\mathscr{H}_{\text{cat-qubit}}\right)$$

$$\mathscr{H}_{\text{cat-qubit}} = \text{span}\{|\bar{\alpha}\rangle, |-\bar{\alpha}\rangle\} \qquad \bar{\alpha} = \pm\sqrt{\frac{2\epsilon_2}{\kappa_2}}$$





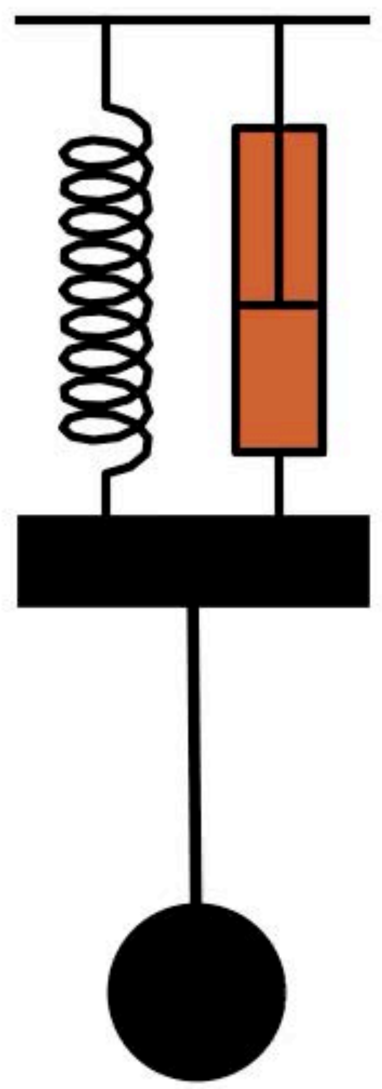
Schrödinger cat state

$$|\bar{\psi}\rangle = \frac{1}{N_+}(|\bar{\alpha}\rangle + |-\bar{\alpha}\rangle)$$

M. Mirrahimi *et al.*, New Journal of Physics, 2014

Cat qubits: how to engineer such a nonlinear dissipation

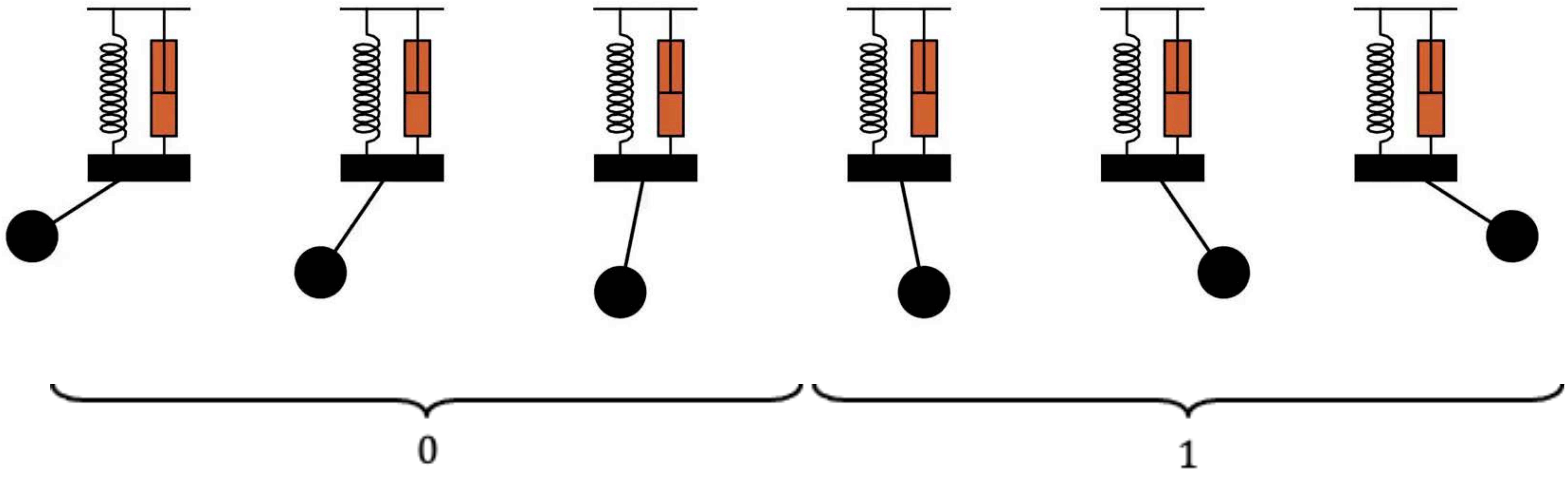
A mechanical analog



Driven damped oscillator coupled to a pendulum

Courtesy of Raphaël Lescanne

Mechanical analog: a bistable system

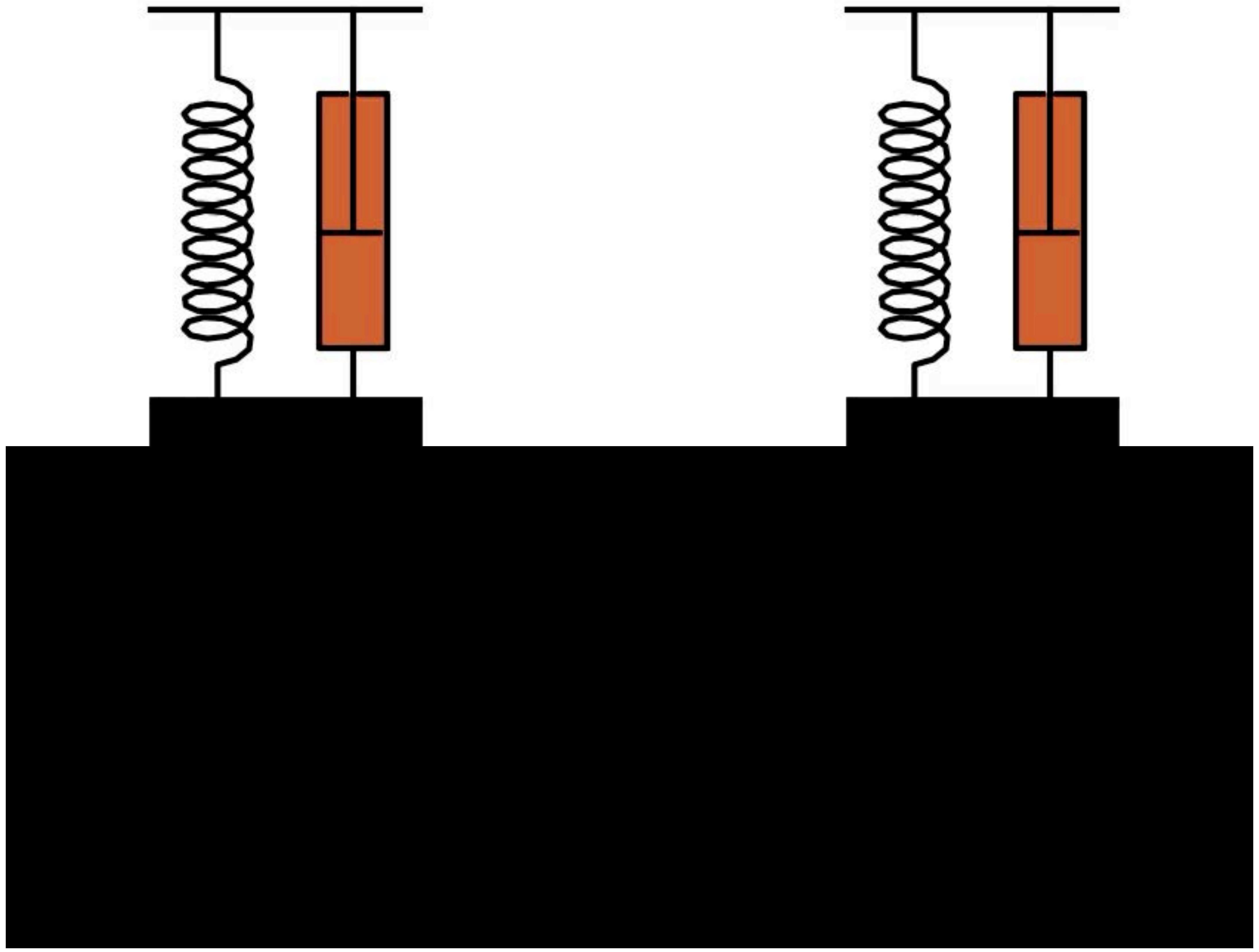


There are two steady states in which we encode information

Courtesy of Raphaël Lescanne

Mechanical analog

Stabilization regardless of the state



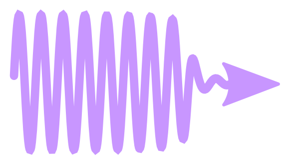
Neither the **drive** not the **dissipation** can **distinguish** between 0 and 1

Important to preserve quantum coherence

Quantum version

Parametric pumping for 2-photon coupling

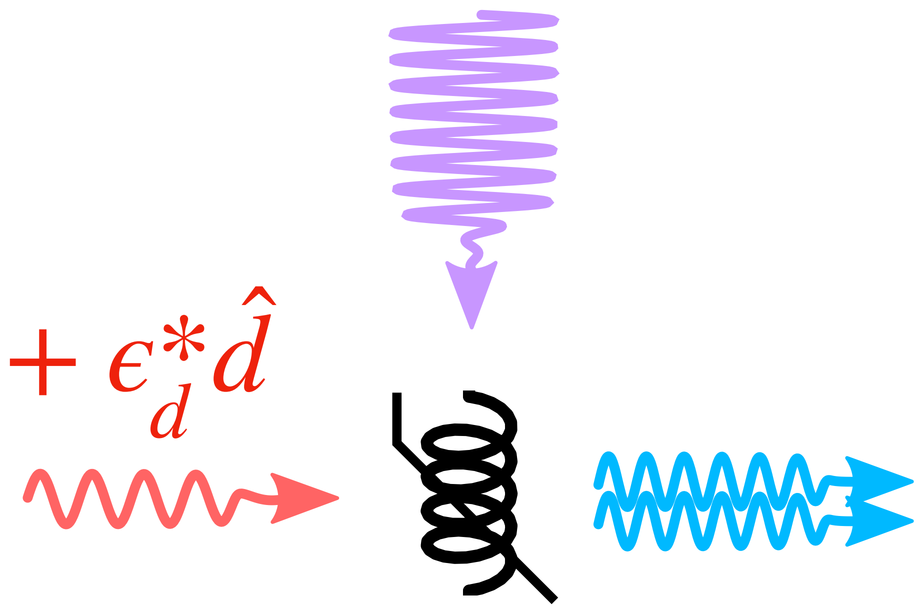
$$\hat{H} = g_2 \hat{a}^{\dagger 2} \hat{d} + g_2 \hat{a}^2 \hat{d}^\dagger$$

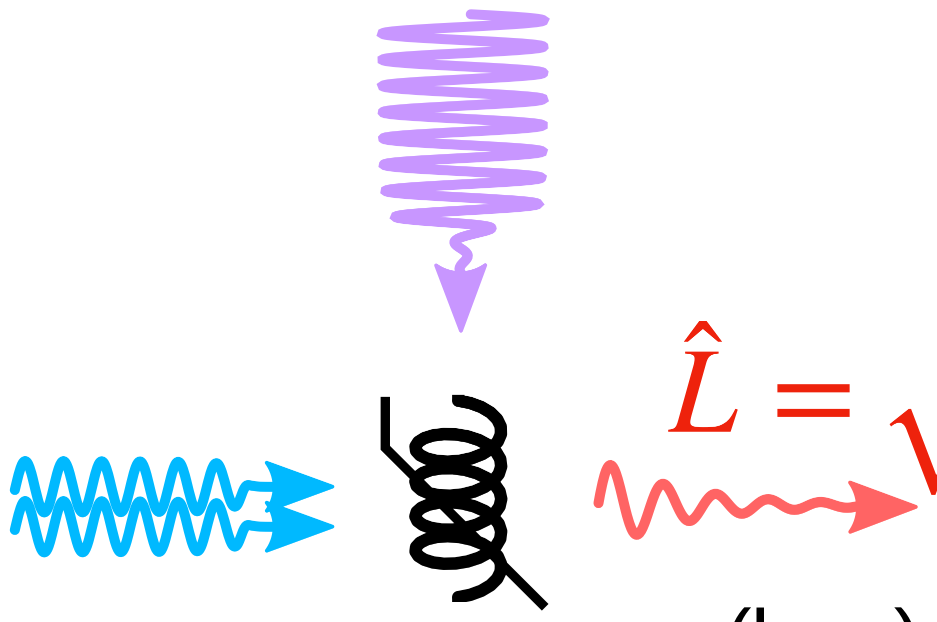


$$\boxed{\omega_p = 2\omega_a - \omega_d}$$

$\hat{H} = \epsilon_d \hat{d}^\dagger + \epsilon_d^* \hat{d}$

(drive)





$\hat{L} = \sqrt{\kappa_d}\hat{d}$

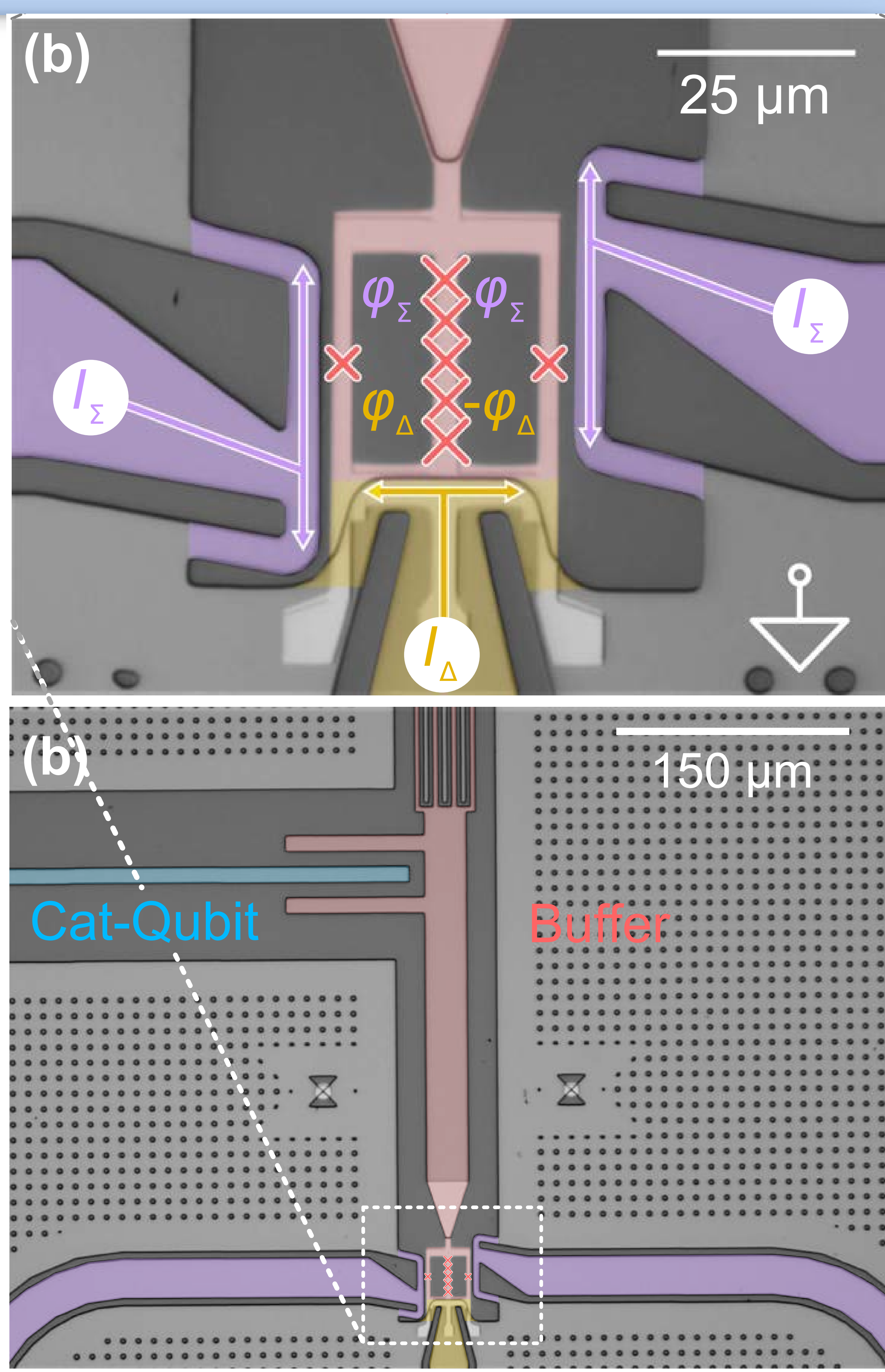
(loss)

$$\boxed{\hat{H}_{eff} = \epsilon_2^* \hat{a}^2 + \epsilon_2 \hat{a}^{\dagger 2}}$$

(two-photon drive)

$$\boxed{\hat{L}_{eff} = \sqrt{\kappa_2}\hat{a}^2}$$

(two-photon loss)



M.M. *et al.*, New Journal of Physics, 2014

Z. Leghtas *et al.*, Science, 2015

R. Lescanne *et al.*, Nature Physics 2020

Mathematical derivation: singular perturbation

Slow/fast dynamics after averaging:

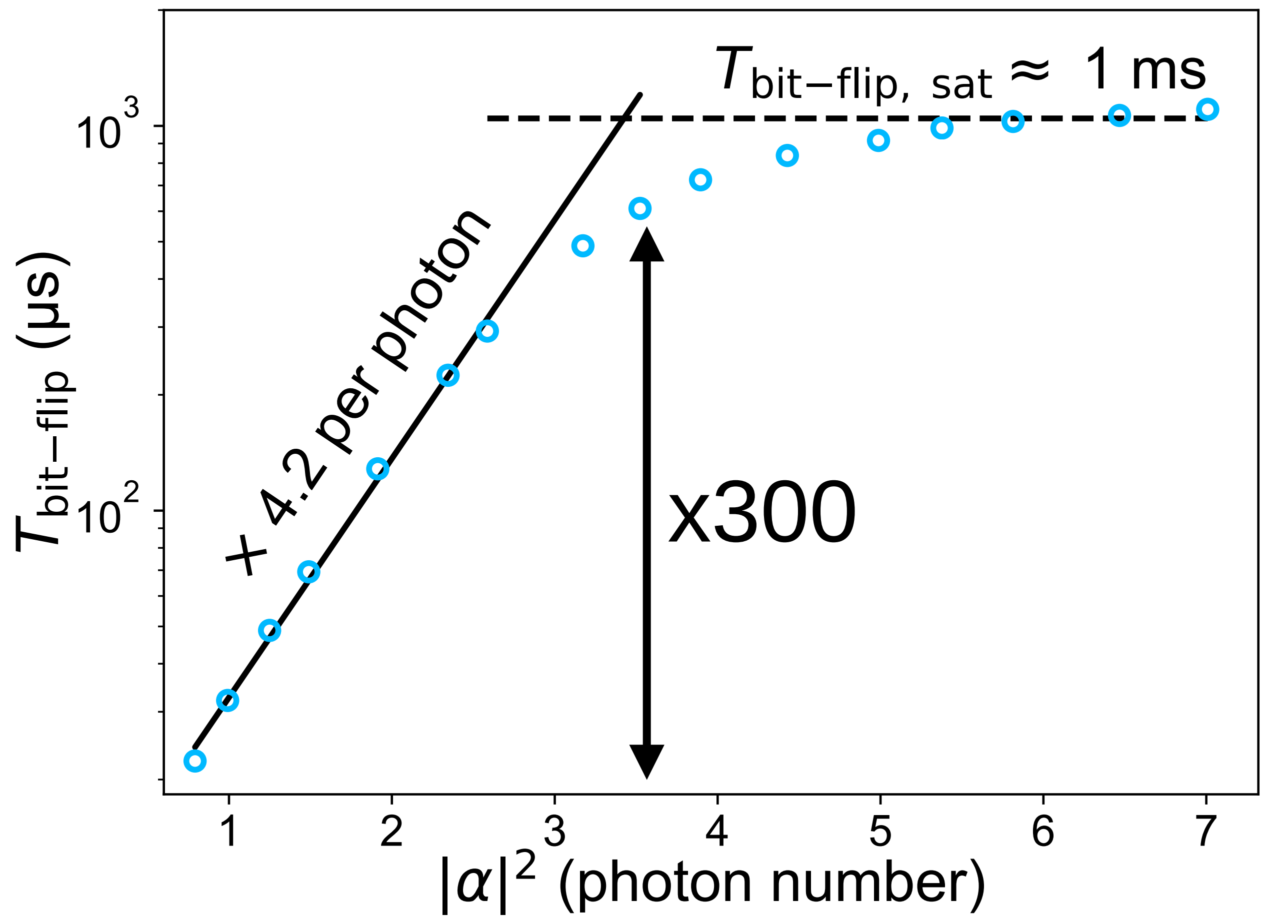
$$\frac{d}{dt}\rho = -i\epsilon\kappa[\hat{H}_{2ph}, \rho] + \kappa(\hat{L}_d\rho\hat{L}_d^\dagger - \hat{L}_d^\dagger\hat{L}_d\rho/2 - \rho\hat{L}_d^\dagger\hat{L}_d/2), \qquad \textbf{in } \mathscr{H}_a \otimes \mathscr{H}_d$$

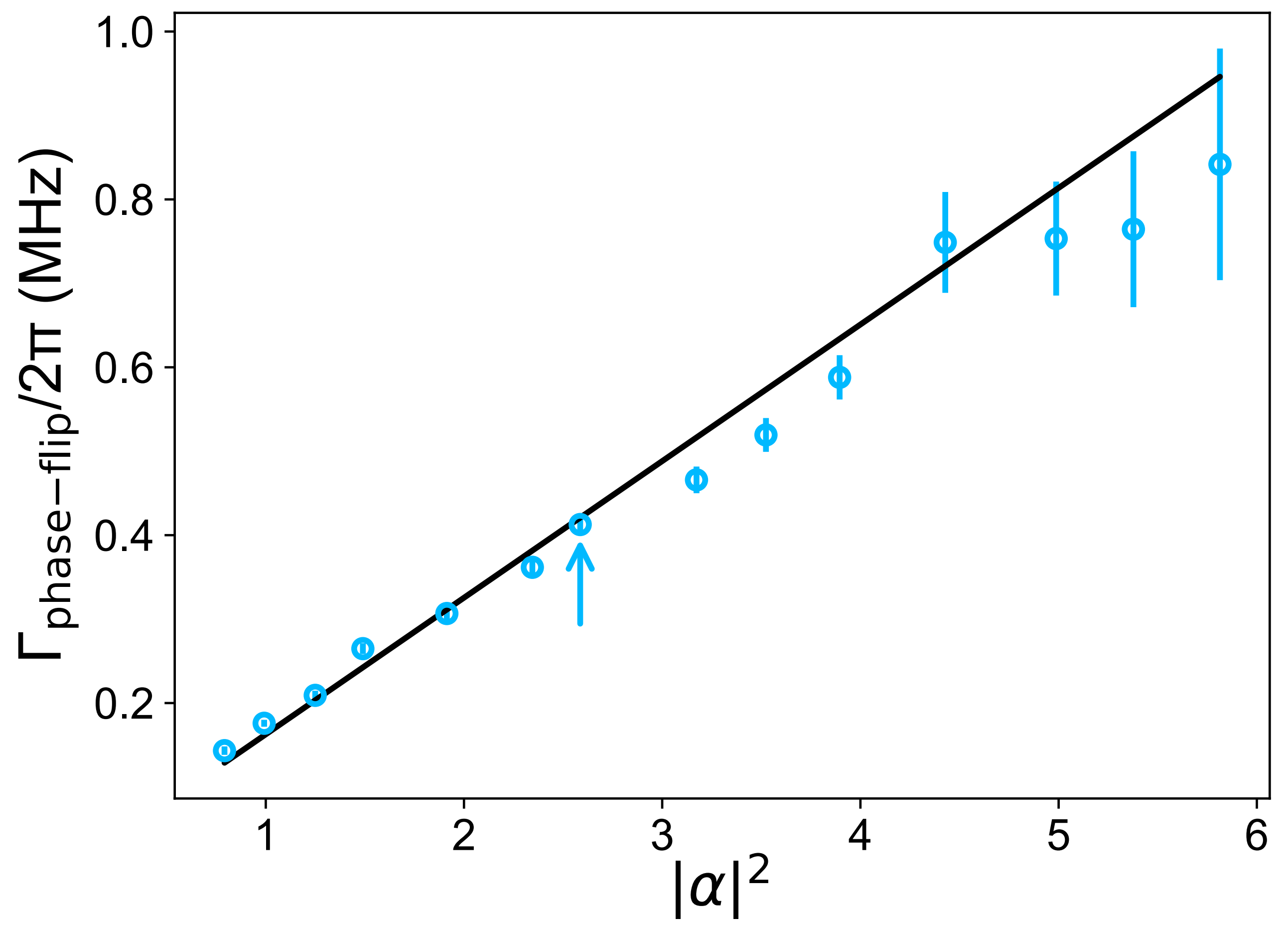
$$\hat{H}_{2ph} = (\hat{a}^{\dagger 2} - \alpha^2\hat{I}_a) \otimes \hat{d}^\dagger + h.c., \qquad \hat{L}_d = \hat{I}_a \otimes \hat{d}$$

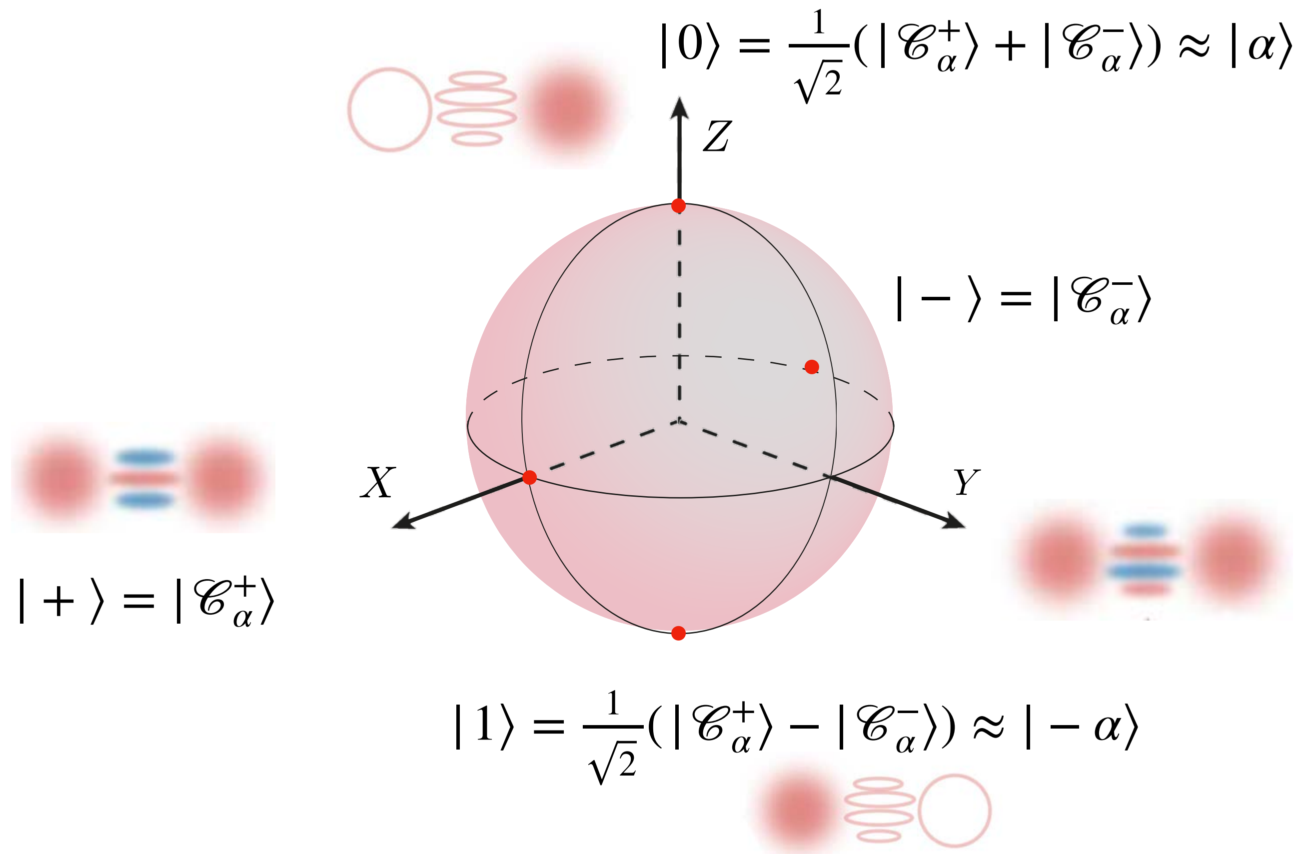
Adiabatic elimination: remove the fast stable dynamics to obtain the effective slow dynamics

$$\frac{d}{dt}\rho_a = 4\epsilon^2\kappa((\hat{L}_{eff}\rho_a\hat{L}_{eff}^\dagger - \hat{L}_{eff}^\dagger\hat{L}_{eff}\rho_a/2 - \rho_a\hat{L}_{eff}^\dagger\hat{L}_{eff}/2), \qquad \hat{L}_{eff} = \hat{a}^2 - \alpha^2\hat{I}_a \qquad \textbf{in } \mathscr{H}_a$$

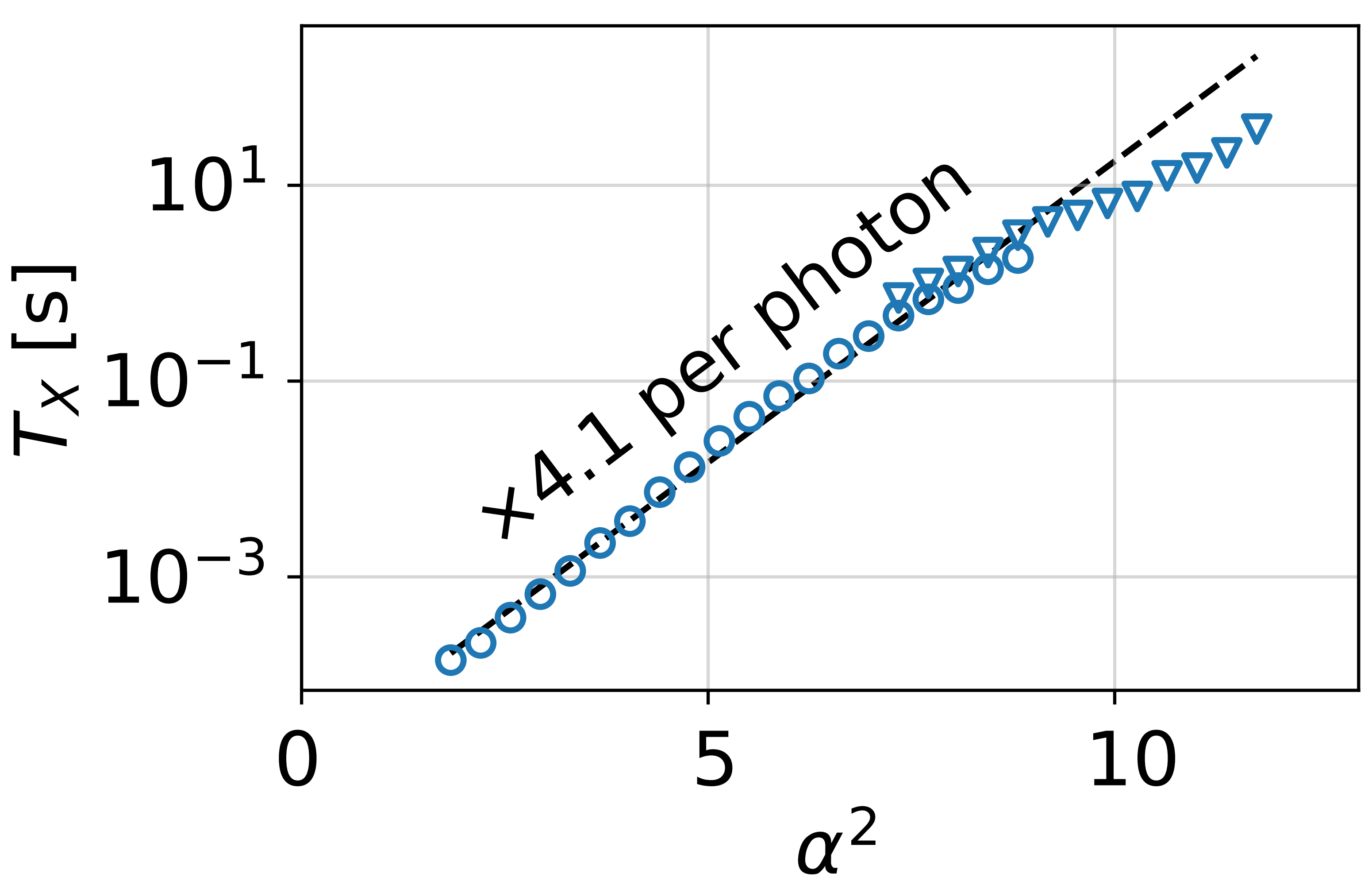
Cat-qubits: exponential protection against bit-flips

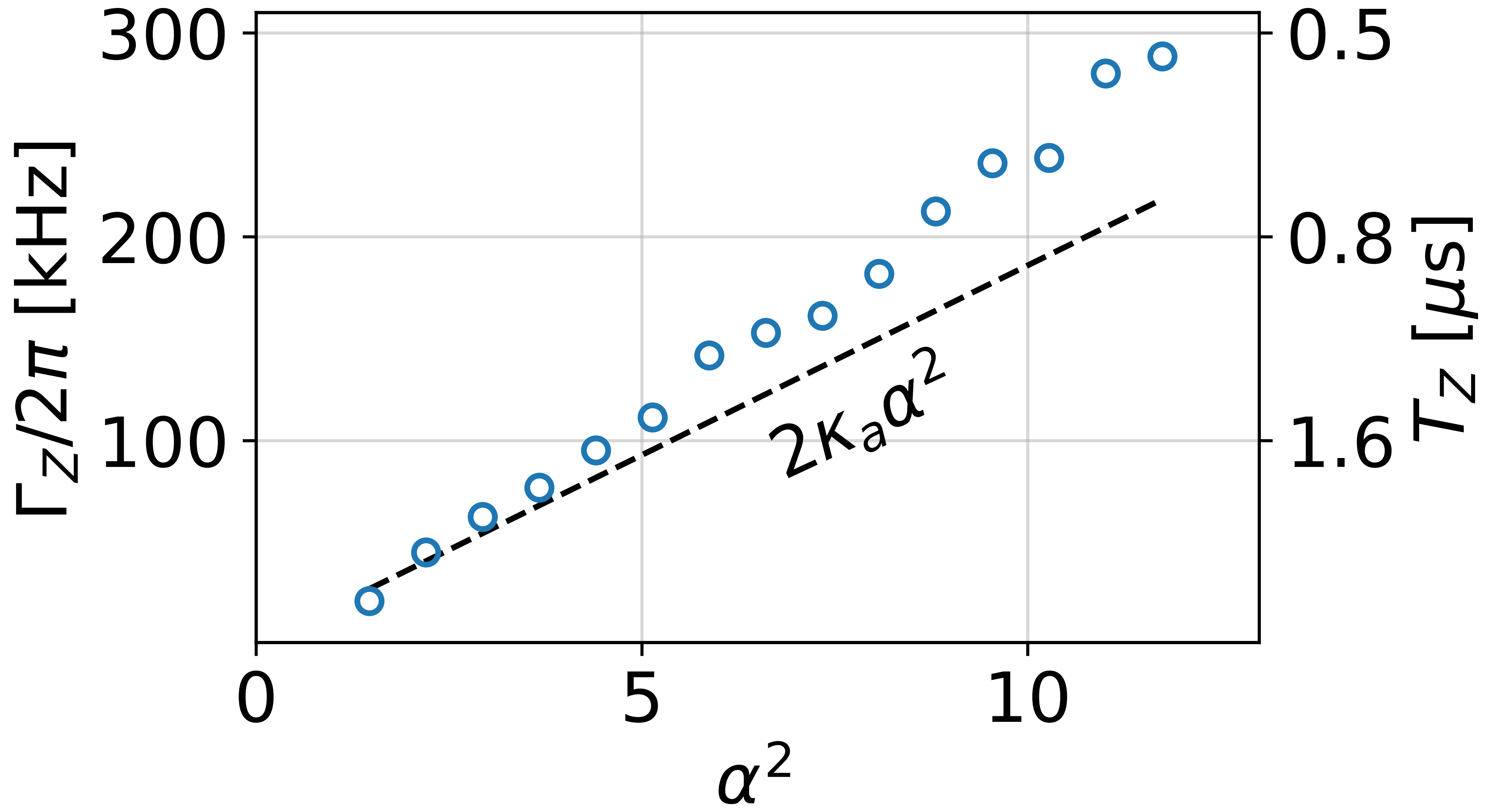






R. Lescanne, Z. Leghtas et al., Nature Physics, 2020





U. Reglade, Z. Leghtas et al., Nature, 2024.

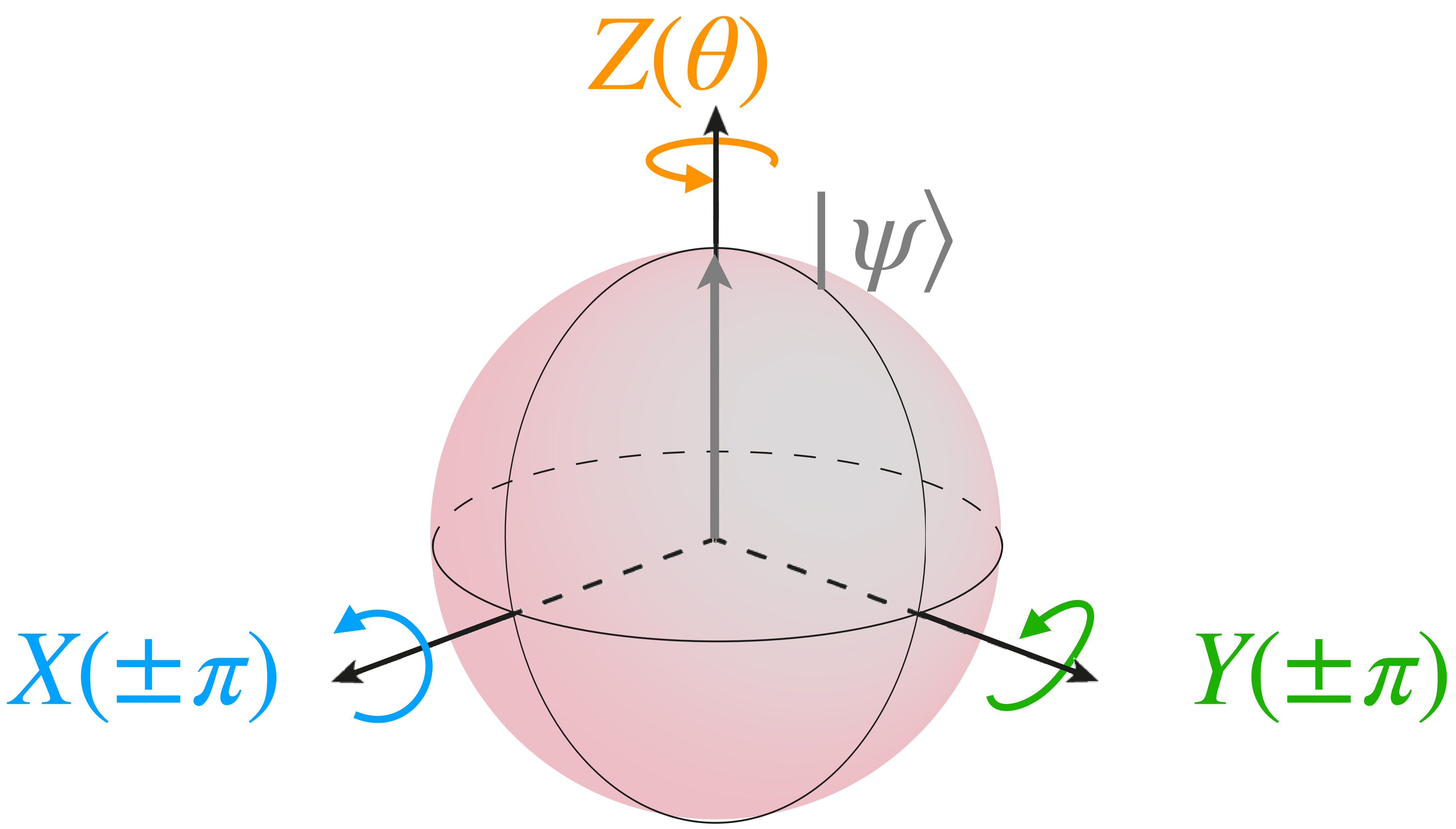
Bias-preserving gates

Definition: A bias-preserving gate preserves the exponential suppression of bit-flips

- A bias-preserving **unitary**

$$\text{1Q} \begin{cases} UZU^\dagger \propto Z \\ \text{Bias-preserving : } \{\pm X,\ \pm Y,\ Z(\theta)\} \\ \text{« Depolarizing » : } U(2) \setminus \{\pm X,\ \pm Y,\ Z(\theta)\} \end{cases}$$

$$HZH^\dagger = X$$



How to do gates?

Quantum Zeno recipe: Hamiltonian control combined with dissipative confinement

$$\frac{d\rho}{dt} = -i[H,\rho] + L\rho L^\dagger - \frac{1}{2}L^\dagger L\rho - \frac{1}{2}\rho L^\dagger L$$

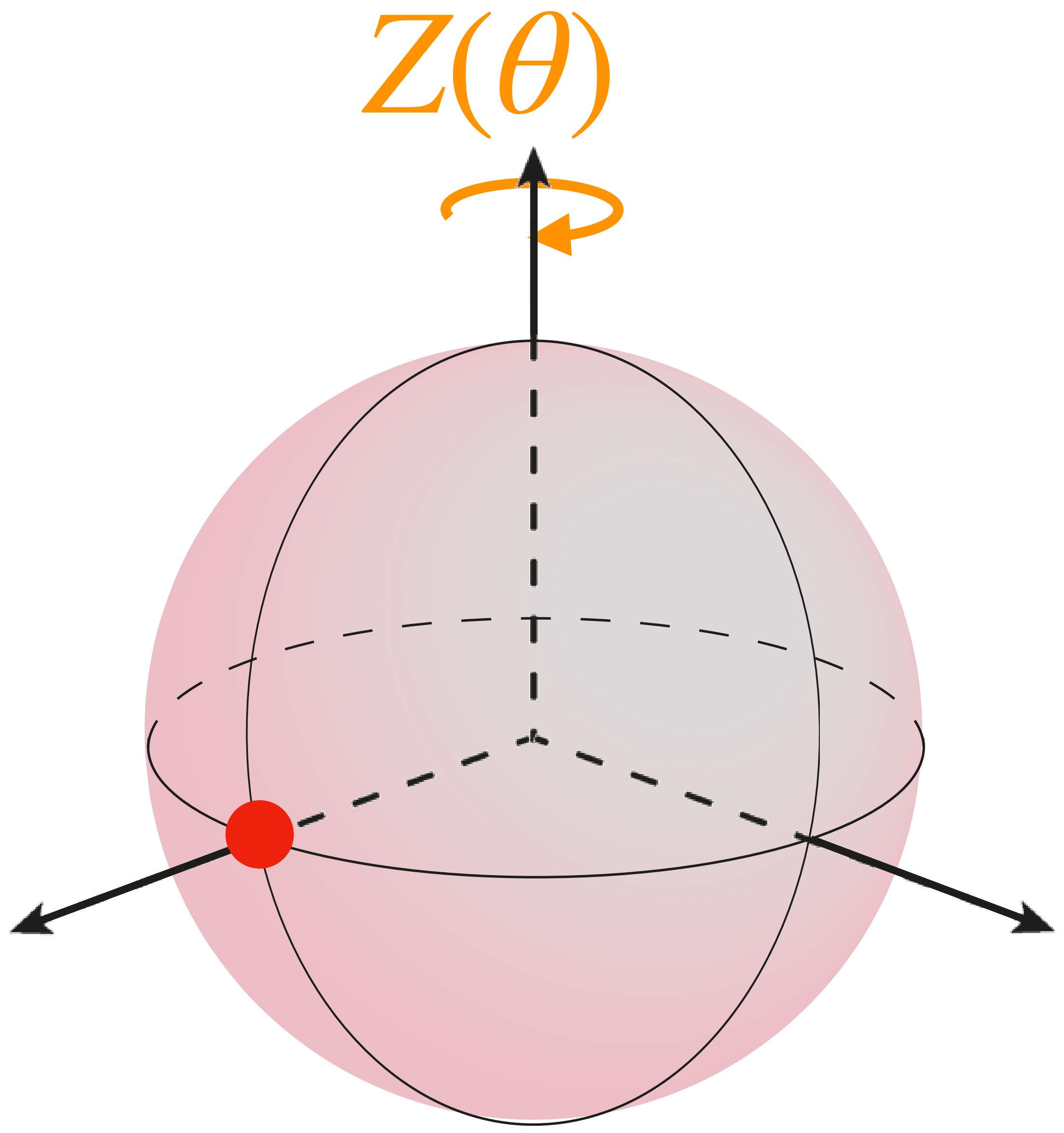
$$L = \sqrt{\kappa_2}\hat{a}^2$$

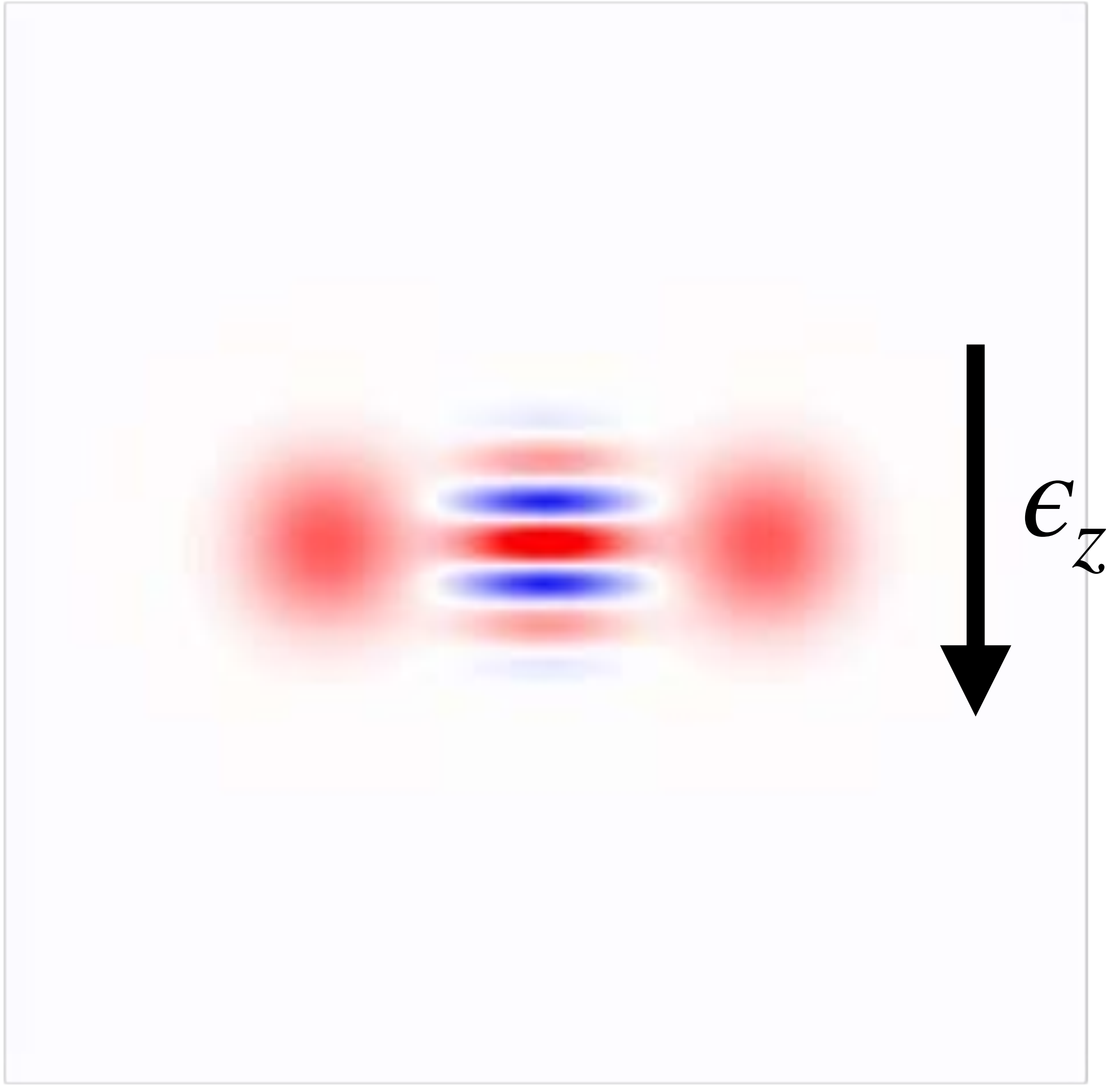
$$H = i(\epsilon_2 \hat{a}^{\dagger 2} - \epsilon_2^* \hat{a}^2)$$

$$\longrightarrow$$

$$H = i(\epsilon_2 \hat{a}^{\dagger 2} - \epsilon_2^* \hat{a}^2) + \epsilon_z(\hat{a} + \hat{a}^\dagger)$$

$$\epsilon_z \ll \kappa_2$$

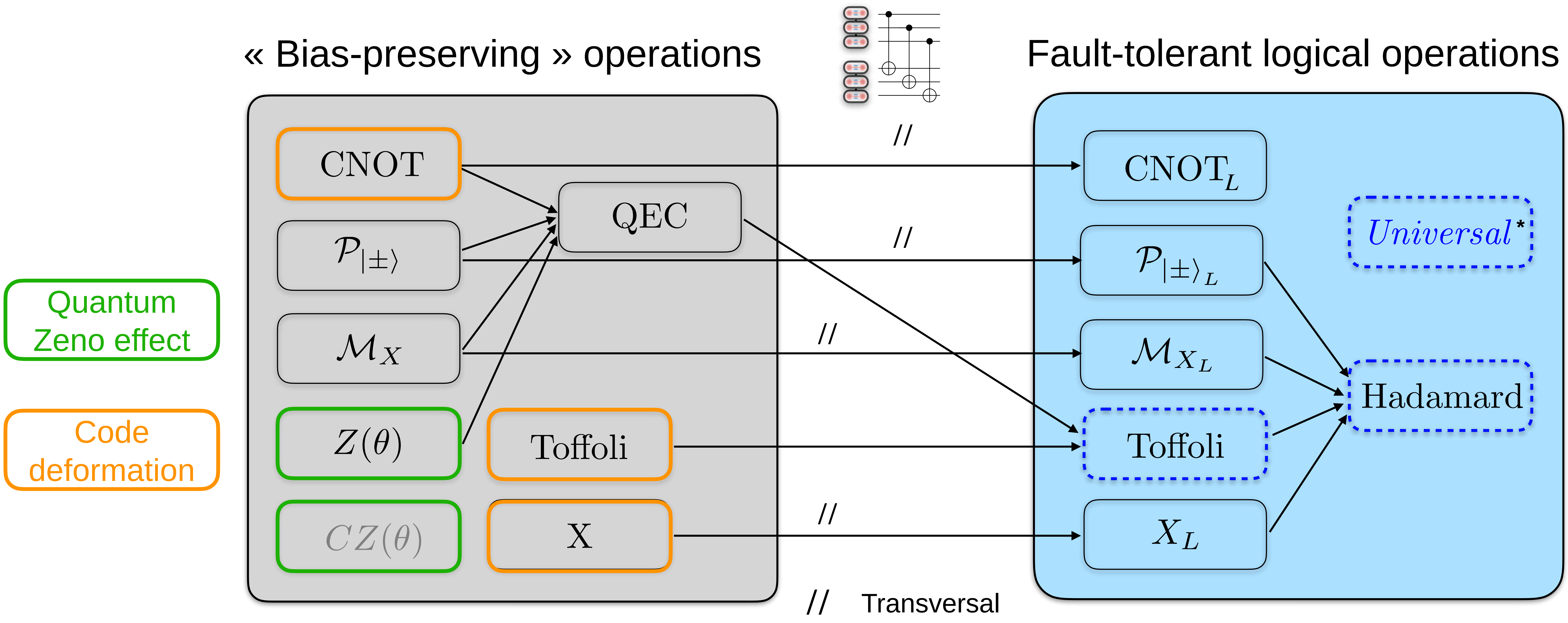




First order perturbation slow/fast system: Unitary gate $Z(\theta)$
Higher order: dephasing + exponentially small bit-flips

M.M. *et al.*, NJP 2014
S. Touzard *et al.*, PRX 2018

Scheme for universal quantum computation

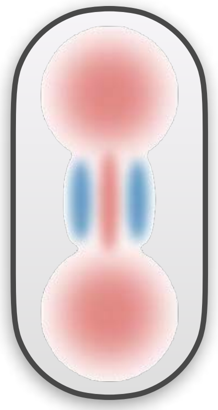


Cat qubit

Repetition cat qubit

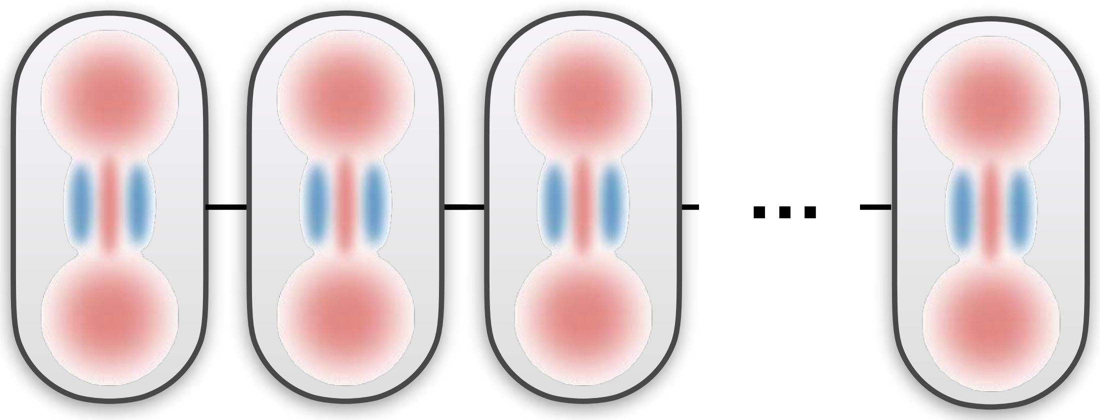
$$|+\rangle = |\mathscr{C}_\alpha^+\rangle$$

$$|-\rangle = |\mathscr{C}_\alpha^-\rangle$$



$$|+\rangle_L = |\mathscr{C}_\alpha^+\rangle^{\otimes n}$$

$$|-\rangle_L = |\mathscr{C}_\alpha^-\rangle^{\otimes n}$$



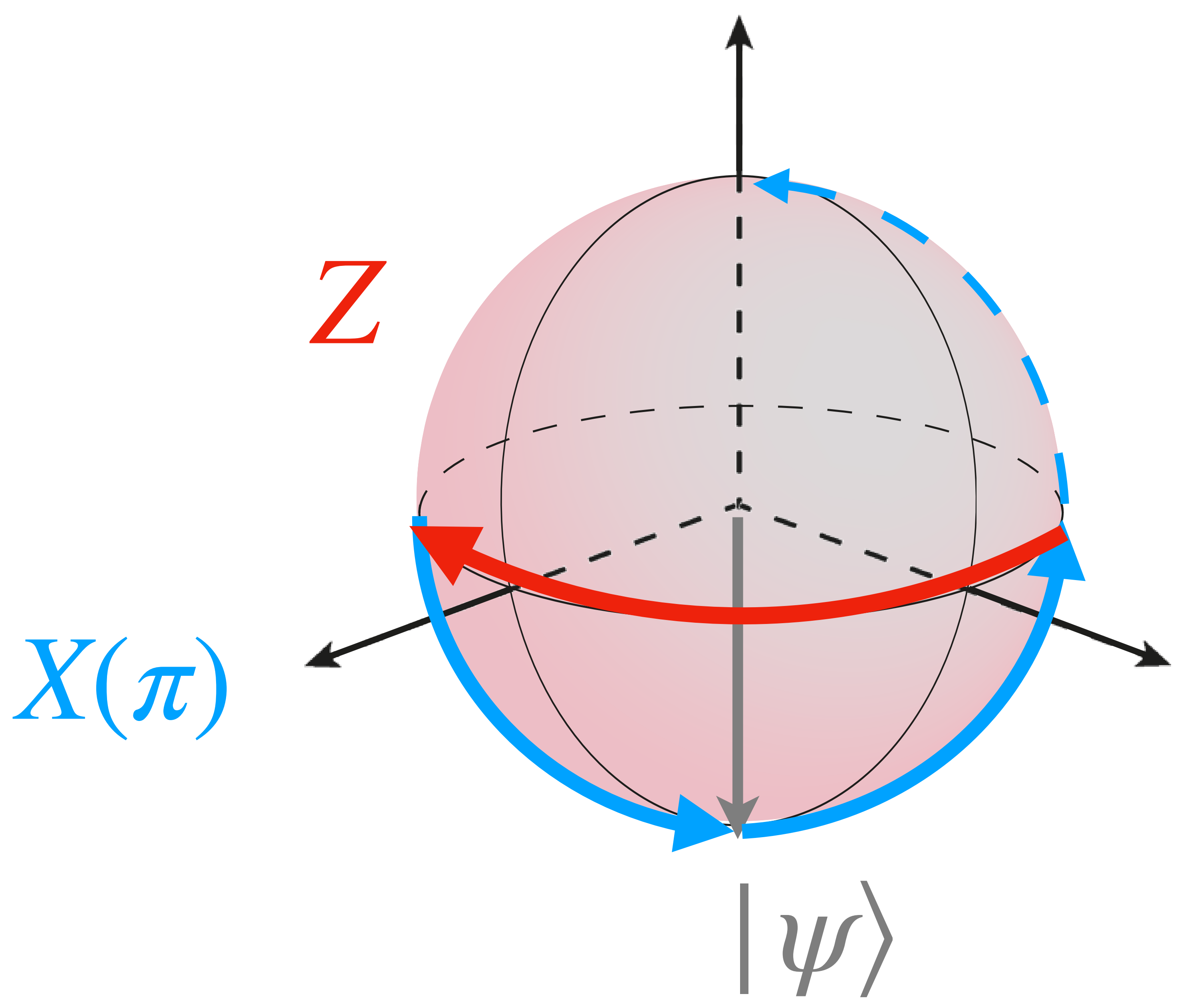
J. Guillaud and M. Mirrahimi, Phys. Rev. X 9, 041053, 2019

Bias-preserving gates

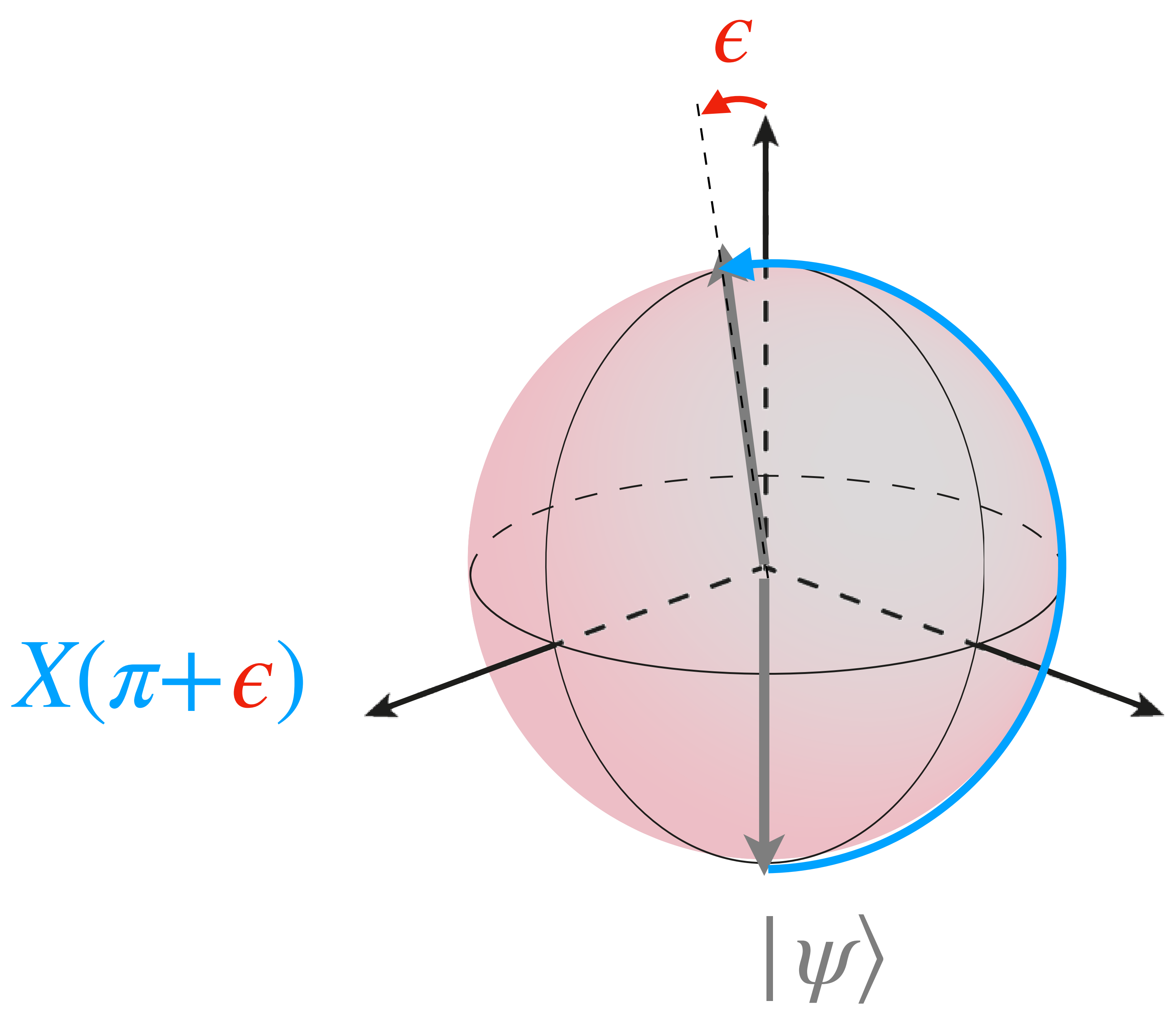
- A bias-preserving **implementation**

$$e^{i\frac{\pi}{2}X} Z e^{i\frac{\pi}{2}X} = X(XZ)$$

$$e^{i(\pi+\epsilon)X} \approx X(I + i\epsilon X)$$



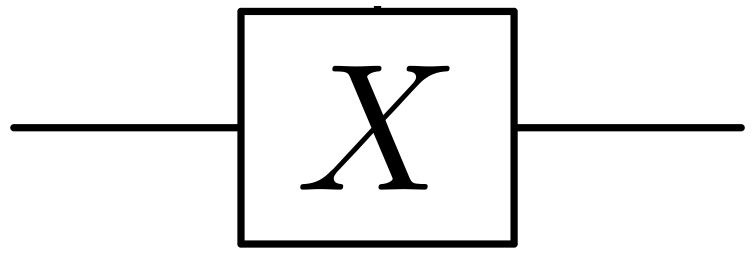
Bias-preserving **continuous process**



Robustness to systematic errors

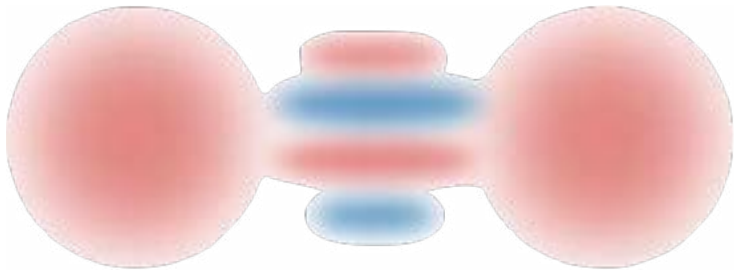
CNOT at the physical (Cat-qubit) level

$$X = |\mathcal{C}_\alpha^+\rangle\langle\mathcal{C}_\alpha^+| - |\mathcal{C}_\alpha^-\rangle\langle\mathcal{C}_\alpha^-| \approx |\alpha\rangle\langle-\alpha| + |-\alpha\rangle\langle\alpha|$$



Control by dissipation:

$$\mathcal{D}[\hat{a}^2 - e^{2i\pi t}\alpha^2]$$



Realization:

Nonlinear interaction with a low-Q dump mode

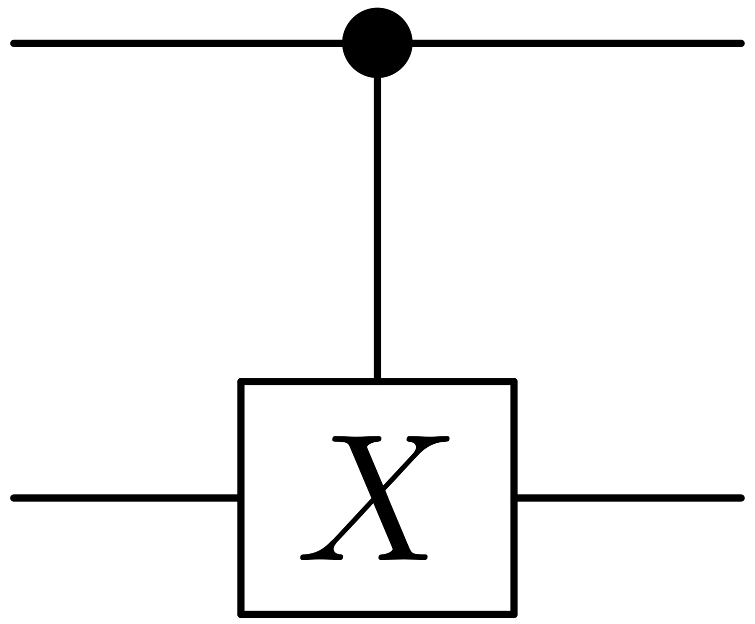
$\hat{H} = \hat{c}^\dagger(\hat{a}^2 - e^{2i\pi t/T}\alpha^2) +$ **h.c.** $\quad\quad \mathcal{D}[\hat{c}]$

TWM + phase & amplitude modulation of the pump

J. Guillaud et M.M., PRX 2019

CNOT at the physical (Cat-qubit) level

$$\textbf{CNOT} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X_2$$

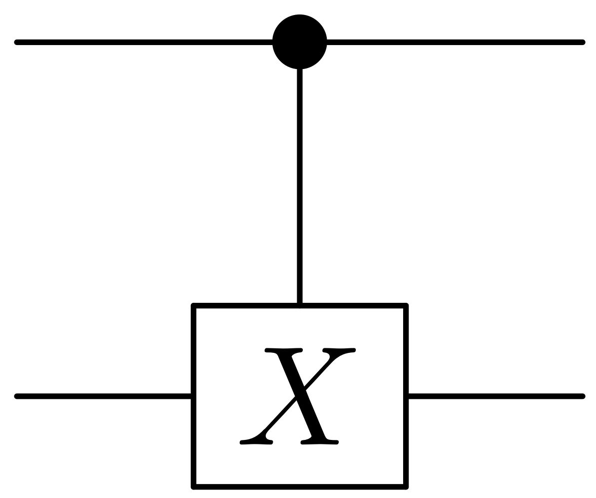


$$|0\rangle = \frac{1}{\sqrt{2}}(|\mathcal{C}_\alpha^+\rangle + |\mathcal{C}_\alpha^-\rangle) \approx |\alpha\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}(|\mathcal{C}_\alpha^+\rangle - |\mathcal{C}_\alpha^-\rangle) \approx |-\alpha\rangle$$

$$\textbf{CNOT} \approx |\alpha\rangle\langle\alpha| \otimes I + |-\alpha\rangle\langle-\alpha| \otimes (|\alpha\rangle\langle-\alpha| + |-\alpha\rangle\langle\alpha|)$$

CNOT at the physical (Cat-qubit) level

CNOT $\approx |\alpha\rangle\langle\alpha| \otimes I + |-\alpha\rangle\langle-\alpha| \otimes (|\alpha\rangle\langle-\alpha| + |-\alpha\rangle\langle\alpha|)$



Control by dissipation:

$$\mathcal{D}[\hat{a}^2 - \alpha^2] \qquad \mathcal{D}[\hat{b}^2 - \frac{\alpha}{2}((\hat{a} + \alpha) - e^{2i\pi t}(\hat{a} - \alpha))]$$

Qubit a in $|0\rangle \approx |\alpha\rangle \rightarrow \mathcal{D}[\hat{b}^2 - \alpha^2]$

Qubit a in $|1\rangle \approx |-\alpha\rangle \rightarrow \mathcal{D}[\hat{b}^2 - (\alpha e^{i\pi t})^2]$

Experimental realization:

$$\hat{H} = \hat{c}^\dagger(\hat{b}^2 - \frac{\alpha}{2}[(\hat{a} + \alpha) - e^{2i\pi t/T}(\hat{a} - \alpha)]) + h.c.$$

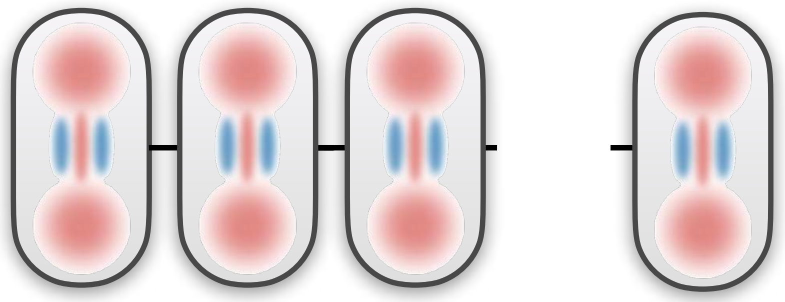
$$+\mathcal{D}[\hat{c}]$$

TWM + phase & amplitude modulation of the pump

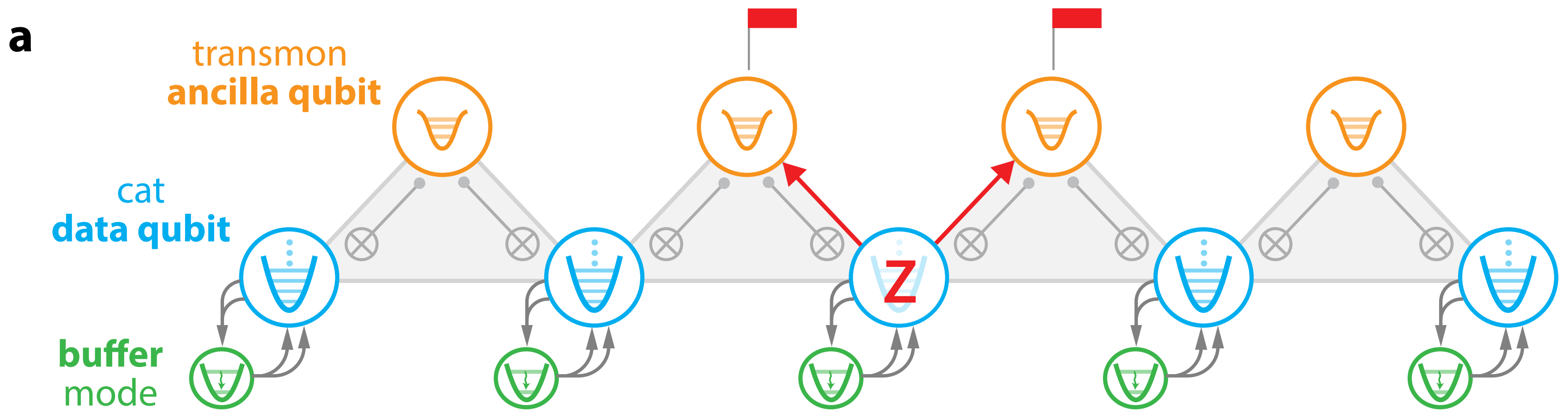
J. Guillaud et M.M., PRX 2019

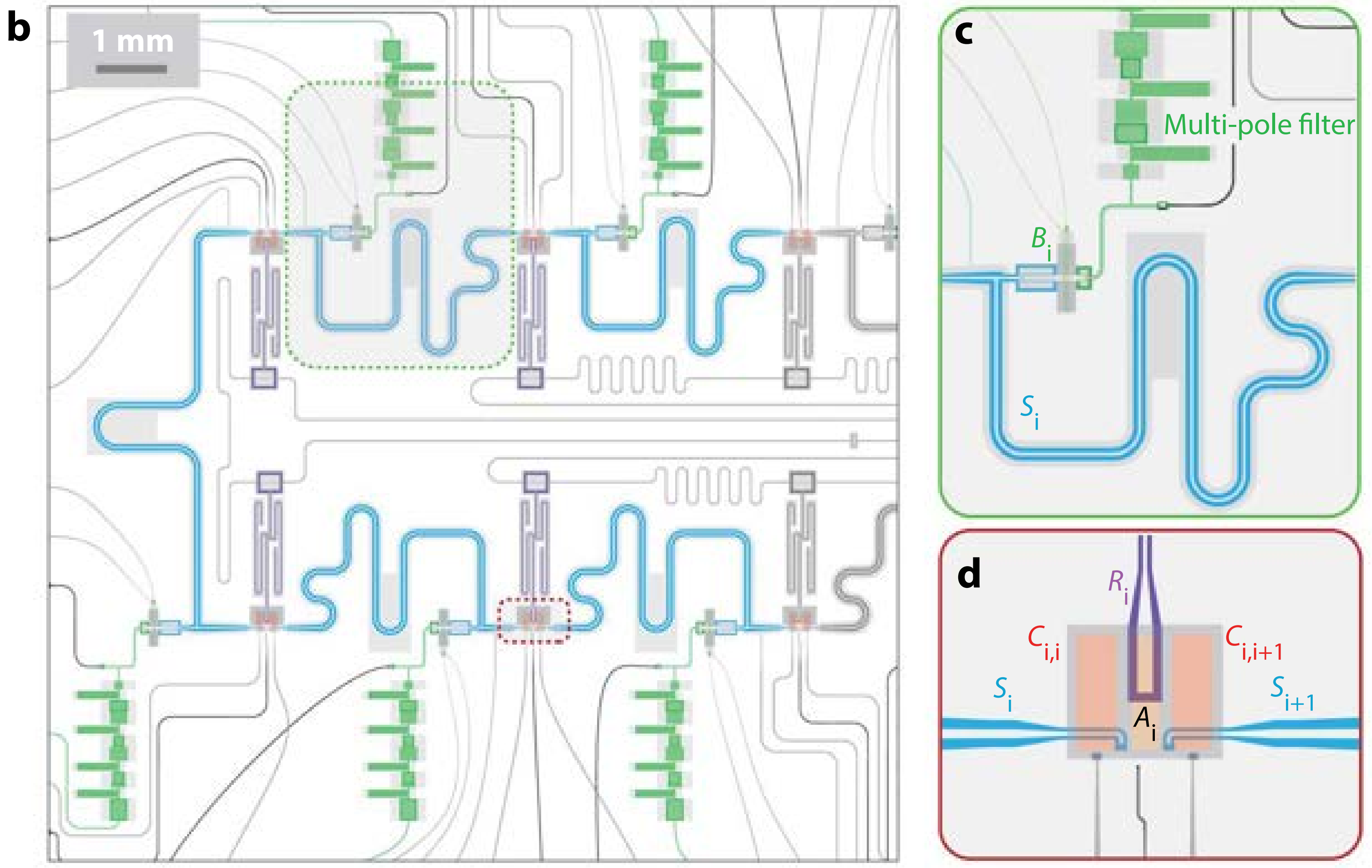
Fault-tolerance roadmap (Alice&Bob, AWS, …)

- Large noise bias regime ($\bar{n} > 10 - 15$ photons): repetition code against phase-flips may be sufficient



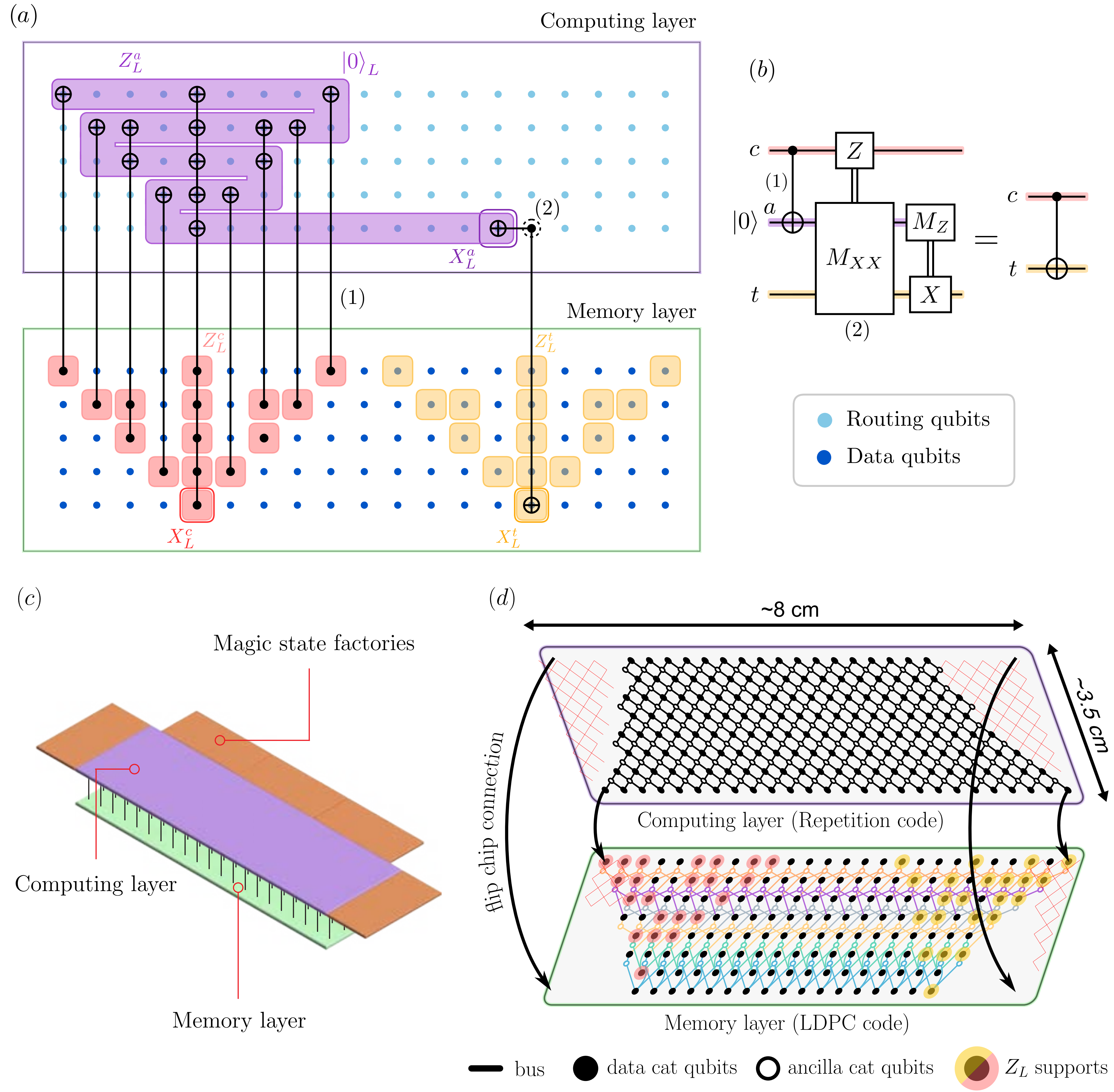
J. Guillaud and M. Mirrahimi, PRX 9, 041053, 2019





AWS experiment: Putterman, Painter et al., Nature, 2025.

2D-local architecture for LDPC-cat qubit concatenation and fault-tolerant operations



FT overhead: 758 cat-qubits with physical error rate 10^{-3} (per qubit, per gate) to encode 100 logical qubits at error rate 10^{-8} (compared to 33700 for surface code)

D. Ruiz *et al.*, Nature Comm., 2025

Thanks!